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THE ANNEALING OF STEEL IN AN ALTERNATING MAGNETIC FIELD.

By H. Pender and R. L. Jones.

THE results obtained may be briefly summarized as follows:

1. It has been shown that the treatment of steel by a cyclicly varying magnetic field during its annealing results in a pronounced alteration of its hysteresis loop. The result is to increase largely the permeability at low and moderate inductions with a corresponding increase of the remanent magnetism. The coercive force and the losses are slightly decreased. The maximum value of the permeability was increased as much as 50 per cent. in some cases.

2. The improvement of magnetic quality depends upon the maximum intensity of the force used in the magnetic treatment and shows an approach to a maximum or saturation value when the force is large.

3. The best maximum temperature at which to apply the cyclic treatment has been identified with the critical point Ar_1, about 690° C., on the iron and steel diagram, and through this fact and the evidence of the micrographic studies, it seems very probable that the good results obtained may be ascribed to a preservation of the fineness of the metallographic structure which steel possesses just after it has passed from the non-magnetic to the magnetic condition.

The above results are incomplete but exceedingly suggestive. They point the way to a new line of research on the treatment of steels for electrical purposes and possibly of those for other uses as well. None but magnetic qualities have been observed, but doubtless mechanical characteristics were also affected.

The possible commercial applications of magnetically annealed steel which suggest themselves are numerous. Many forms of direct current apparatus might be decreased in cost by its employment provided the requisite commercial conditions for the treatment could be obtained.

Thus if the fields of direct-current generators and synchronous motors could be made of the material, the weight of iron could be decreased, though not in so large a proportion as the increase of permeability, together with a corresponding decrease in the mean length per turn of

TABLE OF RESULTS.

Sample No.	Maximum Permeability.	Flux Density (C.G.S. Lines per Sq. Cm.) at which Maximum Permeability Occurs.	Permeability at Given Flux Densities (C.G.S Lines per Sq. Cm.).			Iron Loss at Given Flux Densities (Watts per Pound at 60 Cycles).			Explanatory Notes.
			5,000	10,000	12,000	5,000	10,000	12,000	
A	2,620	6,000	2,580	1,900		.490	1.30		Material A as received.
A_1	12,800	8,400	10,200	11,600		.180	.560	.763	Max. temp. 760° C. $H_{max.}=18.5$ from 760° C. to 340° C.
A_2	8,400	7,200	7,730	6,600		.188	.605	.822	Max. temp. 760° C. Annealed plain.
A_3	13,100	8,500	10,500	11,900	6,350	.222	.630	.837	Max. temp. 870° C. $H_{max.}=18.5$ from 870° C. to 160° C.
A_4	9,000	6,850	8,400	7,500	4,050	.236	.662	.876	Max. temp. 870° C. Annealed plain.
A_5	13,500	7,200	11,500	11,300	6,200	.216	.650	.850	Max. temp. 790° C. $H_{max.}=18.5$ from 655° C. to 300° C.
A_6	7,020	6,100	6,780	4,260	1,830				Test on effect of max. temp. of magnetization.
A_7	8,370	6,250	7,920	4,870	1,200				Max. temp. 805° C. $H_{max.}=18.5$ on A_7 from 500° C., on A_8 from 625°, on A_9 from 750° C.
A_8	10,200	7,000	9,100	6,820	3,000				
A_9	9,620	6,700	8,530	5,570	1,760				A_6 had no winding. Mag. current off at 275° C.
A_{10}	3,200	6,600	3,000	2,200					Max. temp. 625° C. $H_{max.}=18.5$ from 625° to 180°.
B	3,330	6,100	3,250	2,400		.558	1.55		Material B as received.
B_1	11,500	7,500	10,400	8,650		.194	.600	.790	Max. temp. 795° C. $H_{max.}=29.6$ from 795° to 400°.
B_2	8,700	6,600	8,040	5,900		.212	.660	.860	Max. temp. 795° C. Annealed plain.
B_3	7,200	5,600	6,980			.240	.680	.870	Max. temp. 765° C. $H_{max.}=69.2$ from 765° to 485°.
B_4	6,530	5,600	6,360			.240	.680	.870	Max. temp. 765° C. Annealed plain.

Sample No.	Maximum Permeability.	Flux Density (C.G.S. Lines per Sq. Cm.) at which Maximum Permeability Occurs.	Permeability at Given Flux Densities (C.G.S. Lines per Sq. Cm.).			Iron Loss at Given Flux Densities (Watts per Pound at 60 Cycles).			Explanatory Notes.
			5,000	10,000	12,000	5,000	10,000	12,000	
B_5	5,980	5,900	5,860	4,280	2,430				Test of effect of intensity of magnetization. Max. temp. 800° C. At 700°, $H_{max.}=.46$ on B_5, 1.85 on B_6, 5.53 on B_7, and 18.5 on B_8. Mag. current off at about 400° C.
B_6	6,870	6,000	6,720	4,430	2,330				
B_7	7,760	6,700	7,320	5,500	2,880				
B_8	9,300	7,200	8,350	7,300	4,000				
C_1	12,100	9,100	9,200	12,000	9,980	.420	1.28	1.73	Max. temp. 835° C. $H_{max.}=18.5$ from 835° C. to 200° C.
C_2	9,000	8,100	7,600	8,700	7,500	.440	1.32	1.78	Max. temp. 835° C. Annealed plain.
C_1'	10,900	9,700	7,500	10,750	9,500	.400	1.21	1.60	C_1 after aging at 100° C. for 865 hours.
C_2'	8,400	9,200	6,400	8,300	7,350	.438	1.29	1.68	C_2 after aging at 100° C. for 865 hours.

the winding and a consequent increase in copper efficiency. The power factor of transformers, induction motors, and alternating-current series motors could be improved, and the constancy of ratio of instrument transformers could be increased by employing magnetically annealed metal.

DETAILS.

There is very little published information on the annealing of steel for electrical uses, a subject of great commercial importance. The mechanical properties of metals, however, as dependent upon their microscopic structure have been the subject of extensive research. In the case of steel especially, metallographists have much information showing relations between the microscopic structure and grain size, and the tensile strength, elastic limit, and percentage elongation at fracture.[1] The structure of a metal, although it depends partly upon its chemical composition, may be largely modified by heat treatment. The arrangement of the chemical or metallographic constituents and the size of the *grains* in a steel depends partly on the elevation of the maximum temperature above the principal critical point and partly on the rate of cooling. There is practically no information available on the relations between

[1] Howe, H. M., Iron, Steel, and Other Alloys, page 217 et seq.

the electrical and magnetic properties and the *grain* size, and such information as is available connecting them with the metallographic structure is scattered and meager. Hence rules for annealing are qualitative and unsatisfactory.

The use of a magnetic field as a formative force during the structural arrangement of iron particles has not been entirely unsuggested. In the old experiment of Beetz, recalled by Ewing, a small constant magnetic field acted along the axis of the cathode of an electrolytic cell containing a solution of an iron salt. The iron deposited was strongly magnetic and practically saturated. More recently the work of Maurain[1] and of Gans[2] has thrown further light on the subject. The former found that a field of ten to fifteen gausses acting on the cell was sufficient to saturate the electrolytically deposited iron and that the resulting product had a hysteresis loop which was almost a rectangular parallelogram. Gans has also studied the peculiarities in the hysteresis loop of iron electrolytically deposited in a magnetic field.

The fact that a magnetic field might have a similar effect on the structural formation which takes place with the heat treatment of steel appears to have attracted no serious consideration. A single reference to such a possibility occurs in a discussion before the American Institute of Electrical Engineers,[3] where Mr. P. A. N. Winand states that he once tried to magnetize bar magnets by quenching them while under the influence of a steady magnetizing force. He thought that the magnets were slightly better than those obtained by the common method, but decided that the difference was not sufficiently great to warrant fuller investigation.

This paper reports a method of annealing steel by which its permeability may be permanently increased by an amount considerably greater than that possible by heat treatment alone. The method consists in subjecting the metal to a cyclicly varying magnetic force during the period of slow cooling from a temperature above the recalescence point. The result manifests itself by a change in the shape of the hysteresis loop, as well as by an increase in its ordinates. The remanent field is considerably increased and the coercive force slightly decreased, *i. e.*, the loop becomes more erect and slightly narrower.

The work at first undertaken was along the line suggested in the opening

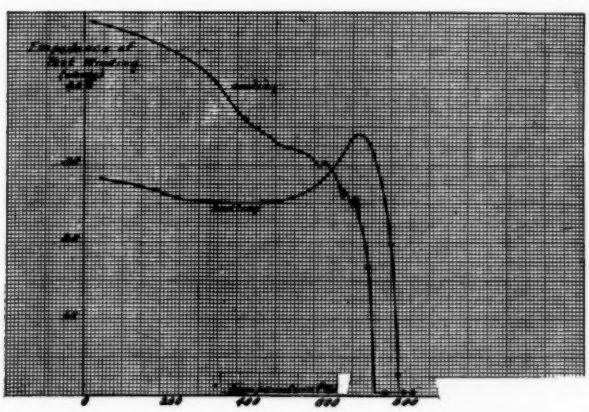
[1] Maurain, C., "Magnetic Deposits Formed in a Magnetic Field," Comptes Rendus, CXXXI., 410 (1900).

[2] Gans, R., "Magnetic Properties of Iron Electrically Deposited in a Magnetic Field," Phys. Zeit., XII., 911 (1911).

[3] Guthe, K. E., "The Influence of Heat Treatment on the Magnetic Properties of Steel and Iron," Trans. A. I. E. E., XIV., 57 (1897).

paragraph. Its plan included a study of the effect of the maximum annealing temperature and of the rate of cooling upon the electric properties and microscopic structure of such silicon steels as are used in electrical manufacture. Early in the work it was desired to have a simple method of detecting the point at which the principal critical temperature or point of magnetic transformation is reached in the heating and cooling.

This temperature, Ar_2 in the equilibrium diagram of iron-carbon alloys, is usually determined by means of the evolution of heat which takes place at that point due to chemical transformations. The fact that a marked change of magnetic permeability also occurs here suggests the use of this phenomenon as a means of the determination. A specimen carrying an asbestos insulated magnetizing winding was placed in the furnace and through the winding a 60-cycle alternating current of 4 amperes constant effective value was maintained during the heating and cooling. The data showing the temperatures and corresponding impedances are shown in Fig. 1. The impedance is approximately pro-



An attempt to determine the transformation point A_2 by a magnetic method.

Fig. 1.

portional to the permeability, since the resistance of the magnetizing winding was low. The plot indicates with a precision of a few degrees the point at which the magnetic property was lost and that at which it was regained. It also has a peculiarity worthy of note.

In the experiments of Hopkinson[1] and Morris[2] on permeability at high

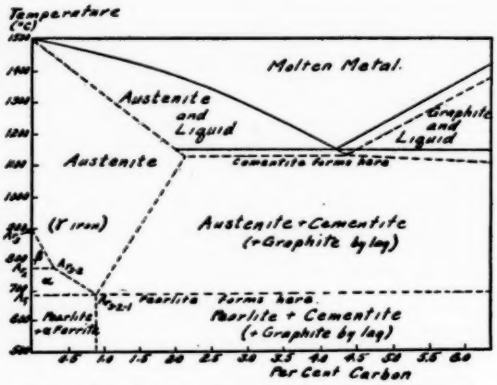
[1] Hopkinson, J., "Magnetic Properties of Iron at High Temperatures," Phil. Trans. (1889), 443.

[2] Morris, D. K., "On the Magnetic Properties and Electrical Resistance of Iron as Dependent upon Temperature," Phil. Mag. (5), XLIV., 213 (1897).

temperatures, all the curves of permeability and temperature approach zero temperature in a general horizontal direction. On the other hand the curve obtained here shows a marked improvement in permeability as the sample cooled to room temperature. Apparently this could be explained only on the assumption that the alternating magnetic field exercised a pronounced influence on the beneficial structural changes which accompanied the annealing, and the circumstance was deemed worthy of further examination. The effect was confirmed by further experiment and this led on to an investigation which precluded the original plan of work. Certain interesting and suggestive results were obtained which this paper aims to present.

METHODS OF STUDY.

Any study of heat treatment may best be based on the equilibrium diagrams of iron alloys. In all commercial steels carbon is an important constituent, and the transformation diagram of the iron-carbon system is the one best known. The work of Guertler and Tamman[1] on the iron-silicon system shows that for all alloys containing less than about 20 per cent. silicon the mass consists of a solid solution of iron silicide in iron with no critical point below 1200°. Hence in spite of the fact that the samples studied are low in carbon, the iron-carbon transformations are



The carbon-iron equilibrium diagram. (Howe.)

Fig. 2.

probably of considerable importance in the structural changes accompanying annealing.

The essential portions of the iron-carbon diagram are reproduced in Fig. 2, which shows transformation temperatures for different percentages of carbon during cooling from the molten condition. Here we

[1] Guertler, W., and Tamman, G., "On the Compounds of Iron with Silicon," Zeit. für Anorg. Chemie, XLVII., 163 (1905).

are interested simply in the left-hand edge of the diagram, since the samples used are all of low carbon content. Above 900° C. the mass consists of austenite, a solid solution of carbon in iron in the gamma form. At about 900° C. the gamma iron changes to the beta form and the austenite passes into a transition stage to which the name martensite has been given. This transformation point is known as Ar_3. That at which the beta allotropic modification changes to the alpha form, about 770° C., is called Ar_2. Alpha iron is magnetic, while beta and gamma iron are non-magnetic. Hence Ar_2 is the magnetic transformation point. Finally, the temperature where the alpha ferrite and the cementite (Fe_3C) begin their segregation to form the composite lamellar structure known as pearlite, is called Ar_1. This temperature is about 690° C. Two transition stages have been distinguished between martensite and pearlite, and to these the names troostite and sorbite have been given. The former is probably the structure of a tempered steel. The sorbitic stage represents the condition where most of the metal has been transformed into ferrite and cementite but where the two have not yet formed the pearlitic structure. It is probably an ultramicroscopic mixture of ferrite and cementite particles. As the cooling continues slowly to room temperatures, the tendency is for the ferrite and cementite to segregate and form interstratified layers of the two materials. This composite pearlite is the characteristic structure of a slowly cooled steel as ordinarily annealed. The microscope is commonly used as an aid to metallographic research and studies have been made of polished sections of several of the samples in investigating the structural transformations and the resulting changes in properties.

The steel used in this research consists of two kinds of silicon alloy sheets such as are used in transformer manufacture. In addition, two samples, C_1 and C_2, of low-carbon dynamo steel were used. Analyses of the former materials as given by the manufacturers are as follows:

	Nos. *A*.	Nos. *B*.
Carbon	.06	.09
Manganese	.13	.17
Phosphorus	.04	.05
Sulphur	.02	.03
Silicon	3.46	3.95

The samples were in the form of hollow squares 3 in. in internal diameter and 5 in. in external diameter. Each sample contained one hundred sheets and weighed about 6½ lbs.

The heat treatment was carried out by means of an electrical resistance

furnace in which several samples could be treated at once. The inside of the furnace consisted of a nine-inch iron pipe closed at the bottom and about 15 in. tall. The walls were ¼ in. thick and the furnace chamber had an iron cover. The furnace pot was first covered with a layer of thin mica sheets. The resistance winding was double, each element consisting of 20 turns of nichrome resistance ribbon (3/16 in. × .005 in.) and was wound on the mica under considerable tension. The use of simply a thin layer of mica to insulate the winding from the furnace walls reduced the temperature gradient, and the form of the resistor material gave a large area of contact on the surface of the furnace. Over the winding about ¼ in. of a paste of magnesium oxide and water was put on and securely bound in place with asbestos tape. The whole was then surrounded with a pipe cover and placed in a sheet iron cylinder 24 in. in diameter, and of about the same height. The space of about 6 in. beneath the furnace and between it and the walls of the container were filled in with ground pipe cover, of the ordinary 85 per cent. magnesia variety. When a run was made the top of the furnace was covered to about 6 in. with the same material.

The furnace took about 2 kw. maximum power. Four specimens could receive heat treatment simultaneously and the heating period necessary was about 8 hours.

The temperatures were measured by means of a platinum-platinum-iridium thermocouple and a Hartmann & Braun recording pyrometer. The thermocouple was enclosed in a quartz tube which was bound to the iron of one of the samples under treatment.

The application of an electromagnetic field to magnetize a specimen during its annealing necessitated that each specimen carry an insulated winding. An asbestos-covered stranded copper wire was used for this purpose. As a further precaution each specimen had its edges covered with folded mica strips and was wound with asbestos tape before receiving its winding. Each winding could be used but once owing to the deterioration of its insulation.

The magnetic testing of the specimens was carried out with the aid of a special testing transformer of which the specimen formed the core. This device had hinged coils, thus expediting tests by making it unnecessary to wind each specimen. Two primary coils of ten turns each enclosed two opposite sides of the hollow square sample and two secondary coils of forty turns each, the other two sides. Permeability measurements were made with direct current, a form of ballistic galvanometer being used to measure the induction.

The hysteresis and eddy current losses were measured at 60 cycles by

supplying an alternating current to the magnetizing winding and measuring the power used in the specimen by means of a wattmeter. A diagram of connections is shown in Fig. 3. The induction was calculated from the induced voltage as obtained from the corrected reading of the voltmeter. The loss measurement was obtained by subtracting from the wattmeter reading the total losses of the secondary circuit. No separation of the hysteresis and eddy current losses has been made.

The generator used as a source of supply was a Mordey inductor alter-

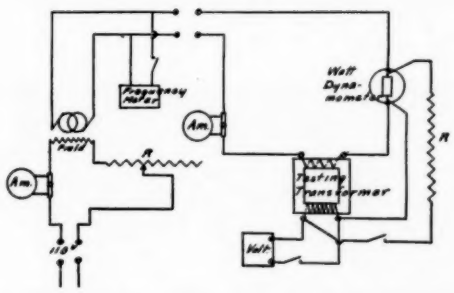


Diagram of connections for magnetic testing with alternating current.

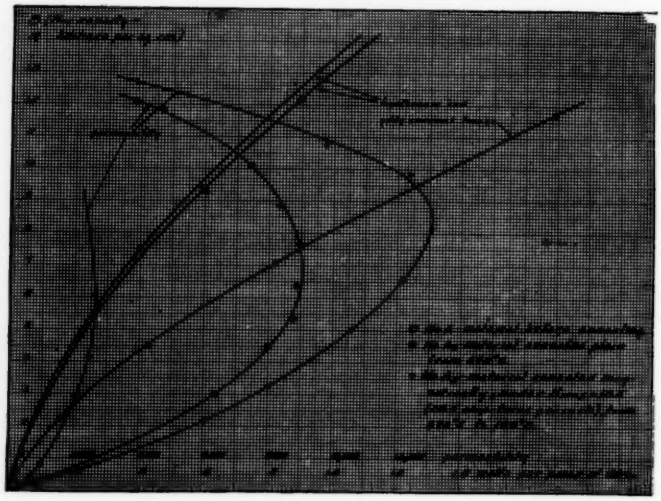
Fig. 3.

nator, giving a wave form which closely approximates a sinusoid. The generator was sufficiently large as to be only lightly loaded by the testing current, and the resistance of the primary circuit was small, the voltage of the alternator being controlled by a large variable resistance in its field circuit. In order to keep the magnetizing current low, specimens of large cross-section, and hence of low reluctance, were used. These conditions all tend to secure a sine wave of induced flux in the specimen under test.

EXPERIMENTAL INVESTIGATION.

In order to confirm the effect of a cyclicly varying magnetic field during annealing, Nos. B_1 and B_2, after being annealed together from 950° C., were prepared for test. The former was wound with asbestos-insulated wire and placed in the furnace with the latter. Both were then heated to 795° C. and slowly cooled. During the cooling B_1 had an alternating magnetic field of maximum value $H = 29.6$ (42.2 ampere-turns per inch) impressed upon it down to 400° C. Upon testing, it showed a maximum value of permeability about 33 per cent. higher than that of B_2 and slightly lower losses. The result was confirmed by a similar run on Nos. A_1 and A_2. A consideration of the method of experiment shows that the only difference in the treatment of the two samples other than that of the magnetic field is the possibility of more even cooling due to the

I^2R loss in the magnetizing winding. As a final test two runs were made in which the unmagnetized sample had a non-inductive winding of the same number of turns as that of the magnetized specimen. The windings of the two specimens were connected in series, and both occupied the same relative positions in the furnace. Under these conditions, the only possible difference in treatment is that due to the magnetizing field. The results of these two tests are those given for Nos. B_3 and B_4 and for Nos. A_3 and A_4. A_3, which was magnetized during its cooling, has a maximum permeability of over 13,000, about 45 per cent. greater than that of No. 8 and slightly lower losses. The permeability and loss



Permeability and loss curves after plain and magnetic annealing.

Fig. 4.

curves of Fig. 4 show the results obtained with A_3 and A_4 and may be taken as typical of these preliminary tests.

In order to determine whether or not as great an improvement in magnetic quality might be looked for in the magnetic annealing of other sorts of steel, two samples of a low carbon dynamo steel were given a treatment similar to that of the other samples. These are Nos. C_1 and C_2. C_1, which was annealed under $H = 18.5$ (26.4 ampere-turns per inch) from 835° C. down to 200° C., showed an increase of permeability over that of C_2 of about the same per cent. as the other samples.

The phenomenon of aging is now of course well recognized in the behavior of iron, and in carbon steels operating under working temperatures an increase of 50 per cent. or more in losses with a corresponding decrease of permeability, is not uncommon. This is due to a gradual

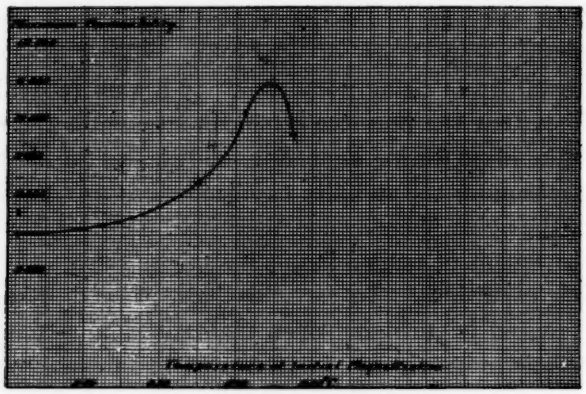
alteration of the structure of the iron caused by the prolonged exposure to the moderate temperature of operation. Silicon steels have the important advantage of being non-aging. It is well recognized among metallographists that the presence of silicon stimulates the transformations which take place during annealing, and in this fact is probably to be found the reason for the stability of silicon steels as regards aging.

The fact of the aging phenomenon suggested, however, that the newly effected improvement of magnetic quality might prove only temporary under working conditions. To test this Nos. A_1 and A_2 and Nos. C_1 and C_2 were placed in an aging oven maintained at a temperature of 100° C. and their magnetic properties tested at intervals over a period of about six weeks. A_1 and A_2 show scarcely any decrease in permeability or increase in losses. In the case of C_1 and C_2 about the usual amount of aging was observed. Each showed a decrease in permeability but the percentage improvement of the magnetically annealed sample was about the same after aging as before. This test indicates that the improvement in properties is just as permanent as that due to ordinary annealing.

Experiments were next undertaken to study the best temperature limits of the alternating magnetic treatment. The method of studying the effect of a maximum temperature at which magnetic treatment is applied, was to place several samples wound with an equal number of turns of asbestos-insulated wire in the furnace and turn the current on the samples at different points on the cooling curve. Thus, after heating a number of samples up to a point a little above the principal critical point, the current would be turned on the first sample at 700° C., on the second at 650° C., on the third at 600° C., and on the fourth at 550° C. When all the samples were in circuit they were connected in series so that the same current flowed through the winding of each. The current was maintained constant by means of an external resistance.

The results shown by Fig. 5 were obtained. They indicate that the best effect of the magnetizing field is to be obtained if the field is impressed at a point on the cooling curve of about 690° C. Referring to the transformation diagram, we see that this temperature seems to correspond to the transformation point Ar_1 where pearlite begins to be formed.

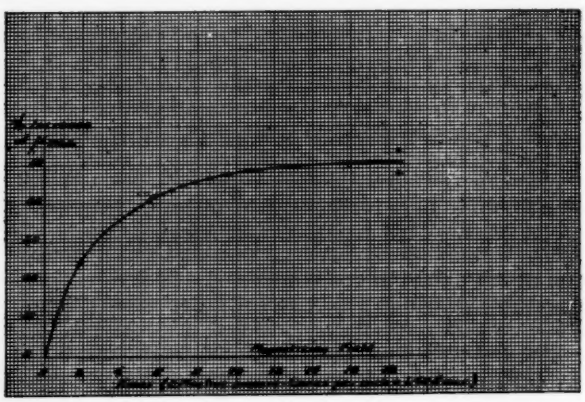
One sample, No. A_{10}, was heated to a temperature of about 600° C. and cooled from that temperature under a magnetizing force of 18.5 (26.4 ampere-turns per inch). Only a very slight improvement in magnetic quality, as compared with that of the other tests, was obtained. This tends to indicate that the critical temperature must be exceeded in order to benefit from the magnetic treatment.



Effect of initial temperature of magnetization on the maximum permeability. Max. temperature of heat = 800° C. Magnetizing force (H max. = 18.5) ceased at 275° C.

Fig. 5.

A study of the effect of various intensities of maximum magnetization yielded the results shown by Fig. 6. The result is what might have been expected from our theories of magnetism and shows that there is a saturation value for the improvement in permeability due to increasingly strong field. This test was made in the following way: A number of samples wound with different numbers of turns and having their windings con-



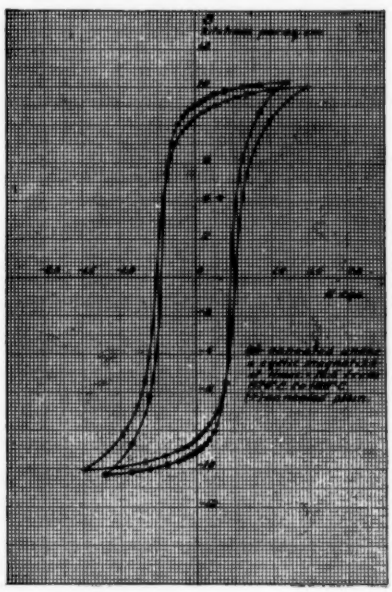
Effect of the magnitude of the annealing magnetizing force on the increase in max. permeability. Max. temperature of heat = 830° C. Magnetizing force on from 750° C. to 200° C.

Fig. 6.

nected in series, were placed in the furnace and after heating to about 800° C. were slowly cooled under a magnetizing force which in the case of each sample was proportional to its turns. The upper two points of the curve were obtained from two samples, treated under as nearly

identical conditions as to heat and cooling as possible (Nos. A_1 and B_8) although their treatment was not simultaneous with that of the others. The curve indicates that under the given conditions of maximum temperature and rate of cooling, the maximum percentage increase of permeability, to be obtained by magnetic treatment over that by plain annealing, is about 50 per cent. This is no indication that a larger improvement is not possible under more favorable conditions of temperature treatment and point of application of the magnetizing force on the cooling curve. However, the same shape of curve for the effect of the strength of the magnetizing field in increasing the permeability would doubtless be obtained.

Such a large increase in permeability as is evidently obtained by appropriate magnetic treatment without a corresponding decrease in eddy-current losses and hysteresis, is somewhat surprising. Hysteresis loops taken with a fluxmeter for samples (1) after plain annealing, (2) after magnetic annealing, are shown by Fig. 7. The effect of the treatment has evidently been to increase the permeability of the silicon steel at moderate and low flux densities without a corresponding increase at the higher densities. It is noticeable that the value of the remanent flux is increased with a slight decrease in the coercive force. Thus although the permeability in moderate fields is much greater after the treatment, the area of the hysteresis loop is only slightly lessened. The effect seems to be rather an increase in the power factor than an improvement in losses.



Effect of magnetic annealing on the shape of the hysteresis loop.

Fig. 7.

The data for Nos. C_1 and C_2 indicate that in the case of a carbon steel the treatment described would result in a larger improvement at higher flux densities than in the case of the silicon steel. A study of the shape of the permeability curves of all the different samples shows that the magnetic treatment shifts the point of maximum permeability toward higher flux densities, thus giving the largest increases in permeability in the range of flux densities used in the design of transformers. In the case of carbon steels the improvement is sufficiently marked, even up to

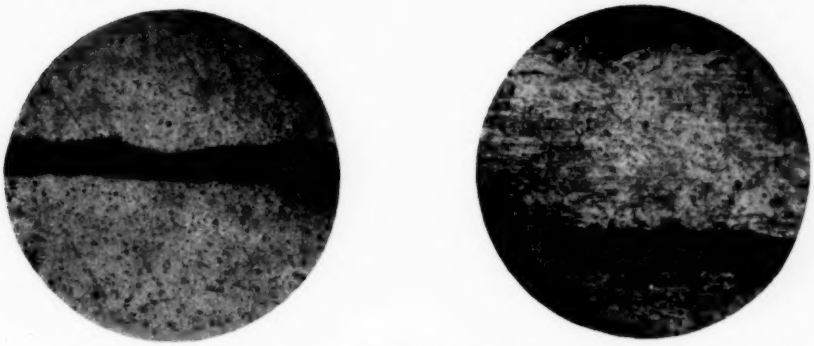
densities such as are used in generating machinery, to be worthy of consideration. The accompanying table gives a summary of the magnetic data obtained.

A micrographic study of several of the specimens has been made in order to see if any connection might be traced between the effect of the magnetic treatment and the metallographic structure. The results of this study are shown by the micrographs of Figs. 8 and 9, which are but two of a number of specimens. Those specimens which were annealed under a magnetizing force seem to show a more homogeneous structure than those which were annealed plain. The micrographs of the etched sections show the grain size of corresponding samples to be about the same and give no further light as to structural difference due to magnetic annealing. The grain of the etched sections has been found, by the investigations of various metallographists, to be a function of the maximum temperature of the heat treatment and the grain structure to undergo a marked change at the transformation point Ar_1.

STRUCTURAL HYPOTHESIS.

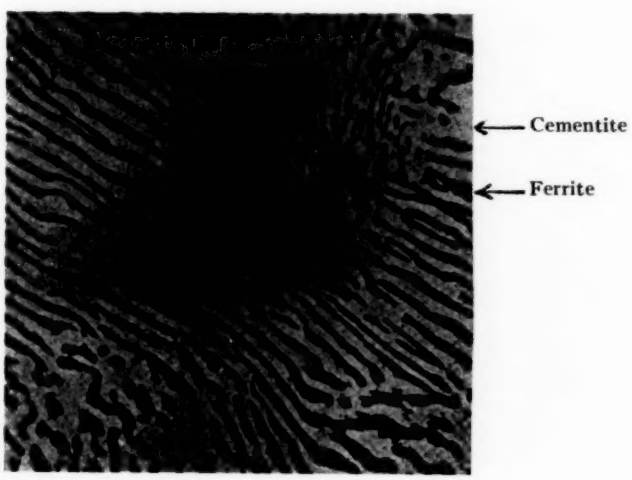
A slight consideration of the molecular theory of magnetism in its relation to the structural forms of steel offers an apparent explanation for the results that have been obtained. Considering the case of a low carbon steel and again referring to the iron and steel diagram, we see that the point Ar_2, where iron regains its magnetic properties on cooling, comes at about 770° C. Just below this point steel shows a remarkably high permeability, especially for low magnetizing forces, and it seems not at all improbable that this characteristic is due to the very fine structure which the sorbitic steel possesses at this point of its heat history. As the cooling progresses further and Ar_1 is reached, the sorbite is gradually transformed into a conglomerate structure containing pearlite with some excess magnetic ferrite. Pearlite is a lamellar structure consisting of curving layers of alpha ferrite and cementite, and under the higher power microscope appears as in Fig. 10. The dark layers of highly magnetic ferrite are partitioned off by interstratified bands of lower permeability cementite. Pearlite always occurs in the constant proportions of about six parts by weight of ferrite to one of cementite.

This structure would seem to divide the material somewhat into molecular groups, each of which has its own arrangement of magnetic molecules. Upon applying a magnetizing force each group has a certain stability of its own and hence the particular field intensity at which the arrangement succumbs to the applied force varies in the different minute groups of ferrite molecules. This characteristic of the pearlite structure shears over and rounds off the hysteresis loop for such a steel, making it



a. Unetched. × 80. *b*. Etched 25 sec. in HNO_3 solution.
No. A_5 heated to 790° C. and magnetically annealed from 655°.
Fig. 8.

a. Unetched. × 80. *b*. Etched 25 sec. in HNO_3 solution.
No. A_3 heated to 870° C. and annealed without magnetization.
Fig. 9.



Pearlite (Osmond).
Fig. 10.

PENDER AND JONES.

less erect than that of the ferrite would probably be, if simply diluted by the cementite present. Hence it may very well be that if we could preserve the more homogeneous sorbitic structure in some way, a very satisfactory electric steel would be the result. The greatest difficulty in the achievement of this result is the fact that practically all the transformations of the iron-carbon system take a considerable length of time and show a large time lag in completing their reactions and physical arrangement. Thus if we were to attempt to quench a steel as sorbite we should probably get some austenite and beta iron remaining, together with some early developed pearlite, and the sample would be likely to prove quite unsatisfactory. Hence slow cooling is quite necessary to the production of a satisfactory electrical steel, as ordinarily this allows the formation of pearlite.

The treatment of the steel by a cyclicly varying field during its cooling probably has the effect of making the structure more homogeneous than would otherwise be the case, preventing the pearlite from forming and keeping the steel largely in the form of sorbite. This it probably does by immediately polarizing the alpha molecules as soon as they are formed by transformation, and thereafter keeping them under control as units of a single magnetic system. Thus they are prevented from forming small secondary groups of stable arrangement.

This suggests the possibility that the peculiar structure of pearlite, always occurring as it does in constant proportions, may very well be due to the magnetic qualities of the molecules of its two constituents and their consequent grouping tendency as soon as the magnetic alpha iron cools out from the beta form.

The results of this investigation seem to lend support to the hypothesis proposed. The most favorable temperature for applying the magnetic treatment was found to be about 690° C., which is about the temperature where a sorbitic steel starts to acquire the structure of pearlite. Then the change in form of the hysteresis loop by the magnetic treatment is precisely what would be expected by the hypothesis. The loop becomes more erect, showing that the steel changes its magnetic condition more as a unit, and thus indicating the greater homogeneity of its structure and the greater uniformity of the type of magnetic groups that prevail throughout. This results in a greater stability of the arrangement acquired under the influence of an externally applied magnetizing force and hence a greater value for the remanent magnetism. This too is confirmed by the hysteresis loops of Fig. 7. The micrographs also lend slight support in that those taken show a finer and more homogeneous structure for the magnetically annealed samples than for those annealed without the alternating field.

CONDITIONS OF SENSIBILITY OF PHOTO-ELECTRIC CELLS WITH ALKALI METALS AND HYDROGEN.

BY J. G. KEMP.

THE object of this investigation is a systematic quantitative study of the conditions of sensibility of photo-electric cells of alkali metals with hydrogen and a determination of the work required to draw an electron out of an atom.

The photo-electric phenomena are very important in connection with the theory of radiation, and for photometric purposes. Planck's theory of radiation requires that the potential due to incident light should increase proportional to the frequency of the light. Moreover, if the theory of the units of energy strikes reality, we should expect that the photo-electric current for very weak intensities of light becomes intermittent, as the electrons are given out only from time to time, so that the phenomenon should resemble the radioactive scintillator. If the intensity of the incident light falls below a critical value, an electron can only escape from the metal, if its kinetic energy is at least equal to the unit of energy received from the beam of light. Photometric measurements have already been carried out by Elster and Geitel, by Richtmeier,[1] by E. L. Nichols and E. Merritt.[2] As the sensitiveness of the photo-electric cells is very great, comparable indeed with the sensitiveness of the selenium cell, we hope to use these photo-electric cells for the measurement of the light from the fixed stars.

The sensitiveness of the photo-electric cells has been increased considerably by the formation of an alkalihydride and by replacing hydrogen by helium. This has been done by Elster and Geitel.[3]

The work necessary to produce an ion can be determined by two independent methods. One method is based on the ionization by α particles, the other on the potential difference which in a discharge tube is necessary to produce ionization by collision. The results of the two methods do not satisfactorily agree with each other. A larger number of determinations of this quantity is necessary before we can explain the difference of the results.

[1] PHYSICAL REVIEW, 29, p. 71, 1909.

[2] PHYSICAL REVIEW, 34, p. 475, 1912.

[3] Physik. Zeitschrift, 11, April, 1910, and August, 1911.

DESCRIPTION OF APPARATUS.

It was required to find the sensitiveness of the photo-electric cells as function of the gas pressure, the electrode distance, the area of metal illuminated, the voltage applied to the electrodes and the intensity of illumination. Fig. 1 shows the glass tube used in making the cell. A spherical bulb, 2.5 cm. in diameter, has two tubes 1.0 cm. in diameter, sealed horizontally and diametrically opposite each other. A vertical tube, 1.8 cm. in diameter, about 15 cm. long is sealed in the top of the bulb.

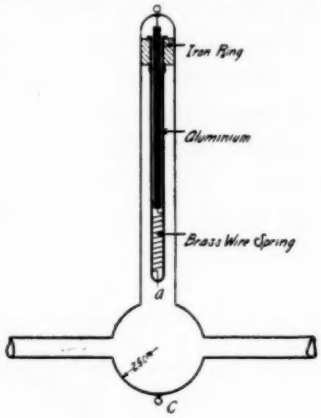


Fig. 1.

At the top of this vertical tube a platinum wire is sealed and fused to an aluminum rod 0.4 cm. in diameter and 10 cm. in length. To the lower end of the aluminum rod is attached a brass spiral spring to which is connected the platinum wire anode. The anode, *a*, being sealed through the lower end of the glass tube which telescopes the aluminum rod. At the upper end of this glass tube is attached an iron ring which fits neatly inside the larger tube. By means of an electromagnet, using a current of 2 amperes, the inner tube carrying the anode, *a*, can be held in any desired position relative to the cathode, *c*, at the bottom of the bulb.

At the bottom of the bulb and diametrically opposite the anode, *a*, is sealed the cathode, *c*, the upper point of which does not extend beyond the surface of the inside of the bulb. In some of the cells the inside of the lower surface of the bulb was silvered, the metal distilled into it and deposited upon the mirror surface. In this way a good contact was insured between the platinum and the metal. In some of the tubes the metal was not distilled into the bulb but poured into it while in the molten state and allowed to solidify over the platinum electrode. The metal in all cases was used as the cathode of the cell. In order to form the sensitive hydride the alkali metal, that is, the cathode, was connected to the negative terminal of a battery of 300 or 400 volts while a resistance of 3,000 ohms and a galvanometer was placed in series with the positive terminal of battery and the anode of the cell. The pressure of the hydrogen gas in the cell was then reduced until the current flowing between anode and cathode caused a faint glow to fill the whole tube. The metal surface, which was very bright before the illumination appeared in the tube, afterward became colored, being brownish for sodium,

bluish violet for potassium, and light greenish for rubidium and cæsium. These colors are due to the formation of a compound of the hydrogen and the metal, which is called a hydride. When the hydrogen is replaced by argon or helium, the cell maintains its high sensitiveness constant for a long time. And even if the metal remains in contact with hydrogen, the sensitiveness did not seem to change during the few days in which the readings were taken.

The distance between the electrodes could be changed by means of an electromagnet, arranged inside the light-tight box, and a cathetometer was used for the accurate determination of the electrode distance.

Fig. 2 shows the entire arrangement of the apparatus for the investigation. The photo-electric cell is enclosed in a light-tight box. Wires connected to the anode and cathode, and to the electromagnet, M, pass

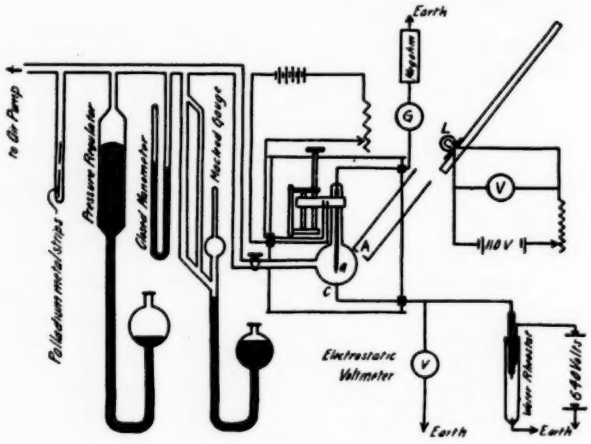


Fig. 2.

through insulators in the walls of the box. The cell is sealed to the system containing a Macleod gauge for measuring pressures up to 0.24 cm., a closed manometer for measuring the higher pressures, a regulator for obtaining small variations in pressure, a tube containing palladium metal strips for supplying pure hydrogen gas, and an air pump The palladium metal was charged with hydrogen gas by the electrolytic method. With an electrolyte of one part of H_2SO_4 and three parts of H_2O, the anode being platinum, and the palladium metal the cathode, hydrogen gas was absorbed by the cathode when 2.5 volts was connected across the electrodes. After charging the palladium, the tube containing it was sealed to the system. When the tube is heated with a small bunsen flame, the metal gives off pure hydrogen gas. The whole glass system, when the pump was cut off, could be filled to a pressure of about 25 cm. when the palladium was heated.

The galvanometer used is a Leeds and Northrup type HS, the sensibility being 3.78×10^{-10} amp. per mm. Deflection for 2 meters scale distance. The anode of the cell was connected to the earth through the galvanometer and a megohm resistance. The cathode of the cell was connected to a variable point in a water rheostat which is in series with about 640 volts from a storage battery. The voltage applied to cathode could be varied by means of the water rheostat and it was measured by a Kelvin electrostatic voltmeter reading 0–600 volts.

The variation of the area of the metal illuminated was obtained by varying the opening of the iris diaphragm A, which is placed at the lower end of a brass tube. This tube was blackened on the inside to prevent reflection of light.

By moving the lamp on the guide the intensity of illumination on the metal in the photo-electric cell could be varied, and this variation calculated directly by means of the inverse square law.

METHOD.

Since the most sensitive conditions for the photo-electric effect are being sought, it is necessary to study the effect due to varying all the possible conditions in order to find the most effective set of conditions. The variables in this work are the following: P the pressure of the gas, D the distance between the electrodes, V the potential difference applied to electrodes, A the area of metal illuminated, L the intensity of illumination, t the temperature of the cell, and d the galvanometer deflection which is proportional to the current.

Two sets of readings are possible for each cell, namely, before forming and after forming the hydride surface. In this paper this process will be called forming.

Four cells were studied: one with cæsium and three with potassium metal. The readings were taken in the order as follows: With t, L, A, P and D constant the values of the deflections, d, of the galvanometer were read for increasing values of V. Thus values of current and voltage were obtained for an ionization curve. This was repeated for three and in some cases four distances of D.

From the above data four ionization curves are obtained which show the effect of varying the distance between the electrodes for constant values of t, L, A and P. If the above three or four ionization curves be called a set, then it is possible to get as many sets as there are values of P, the gas pressure. From three to five different values of P were selected for each cell and in this way the effect due to change of pressure was studied.

Ionization curves were also obtained in which A, L, P and D are constant for three or four different temperatures. After the forming process similar sets of ionization curves were taken except those for temperature changes. In addition to the ionization curves taken after forming the cell No. 4, sets of data were taken in which L, V, P, t and D are constant while A and the current varied. Also readings were taken for A, V, P, t and D constant while L and the current varied.

A total of thirty-six plates, each containing four or five curves were obtained for the four cells. On account of the similarity of the large number of curves and data taken only representative curves will be given for potassium cell number four.

CURVES.

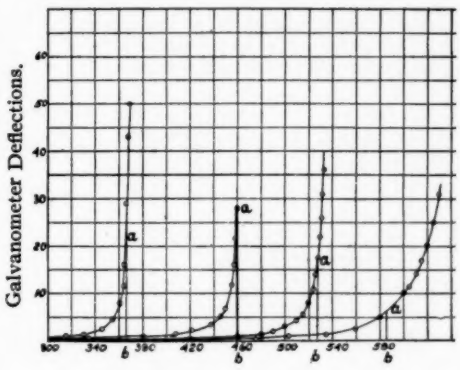
A in cm^2. represents the area of metal illuminated.
L in candle feet represents the intensity of illumination.
t in ° C. the temperature of the cell.
P in mm. mercury represents the pressure of the hydrogen gas.
D in cm. represents the distance between the electrodes.
V in volts represents the potential difference between electrodes.
d in mm. represents the galvanometer deflections.

CRITICAL VOLTAGE AND CURRENT.

It was found that when the voltage was applied to the cells, it could be increased to a certain definite maximum value before a deflection of the galvanometer was noticeable when the light was not acting. If this voltage was exceeded by an amount hardly readable on the voltmeter a deflection of the galvanometer resulted. Furthermore, if the light was permitted to act and the voltage applied, equal to the maximum value determined as stated above, a definite deflection of the galvanometer was produced; and, when the light was suddenly turned off the galvanometer deflection always became zero. However, if this maximum value of the voltage were exceeded and the light turned off, the deflection of the galvanometer was decreased but never became zero.

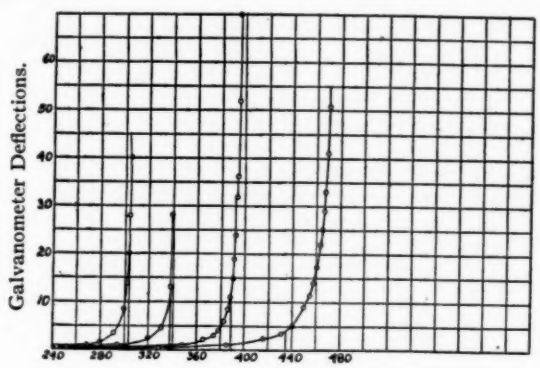
This voltage, therefore, represents the maximum which may be applied to the cell, in this particular case, and at the same time have the ionization current produced only by the action of light. This value of the voltage I shall call the "critical voltage," and the corresponding current the "critical current" for this particular condition of the photo-electric cell. The critical voltage and the critical current taken together give a definite criterion for determining the best conditions for sensitiveness. When the critical current is a maximum and the critical voltage is a

minimum, then the most sensitive conditions obtain. Therefore, the critical voltage and the critical current are given in the data and shown by the vertical lines as for example *ab* in Fig. 3. If the voltage is increased beyond the critical value, when the cell is in the dark, the current will increase suddenly, so that it cannot be measured with the galvanometer. This indicates that the sparking voltage is not much greater than the critical voltage. Some physical results, chosen from a large number of observations are given in Figs. 3 to 17. Four groups of curves will be given. In the first group, Figs. 3 to 10, relating to a potassium cell before formation of the hydride, the two most important independent variables, the electrode distance and the gas pressure are studied. The same holds for the second group of curves, Figs. 11–15, which however have been taken after the formation of the hydride. The variation of the area illuminated is represented in Fig. 16, and the variation of the intensity of illumination in Fig. 17.



Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Gas pressure 5 mm. Before forming.

Fig. 3.

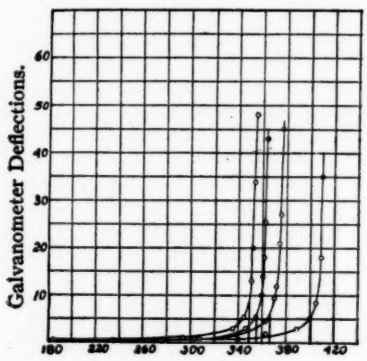


Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Gas pressure 2 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 26°$ C.

Fig. 4.

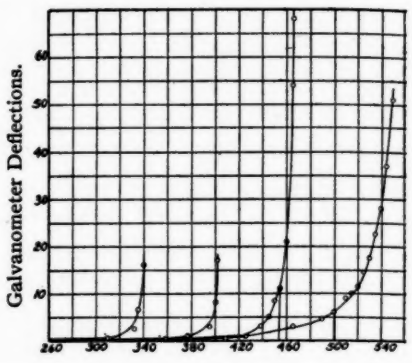
The following remarks may be made on curves Figs. 3–10. The illuminating voltages are in the order of the values of D.

Fig. 3. The critical voltage for $D = 0.5$ cm. is the smallest, while the critical current for $D = 1.0$ cm. is the largest; therefore, the best condition for this pressure is for some value of D between 1 and 2 cm.



Variation of electrode distance reading from left to right 1–0.5–2–3 cm. Gas pressure 1 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 26°$ C.

Fig. 5.

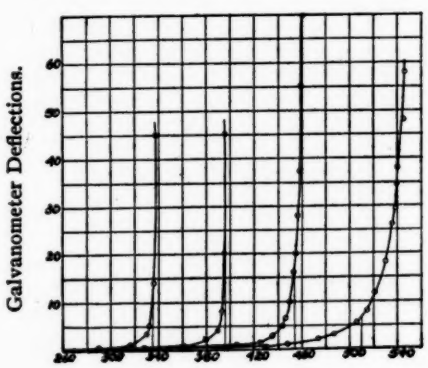


Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Temp. salt and ice bath. Gas pressure 3 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 20°$ C.

Fig. 6.

Fig. 4. The order of the illuminating voltages is the same as that for values of D. The critical voltage for $D = 0.5$ cm. is the smallest and the critical current is the largest; therefore, the best conditions for sensitiveness are shown by this curve.

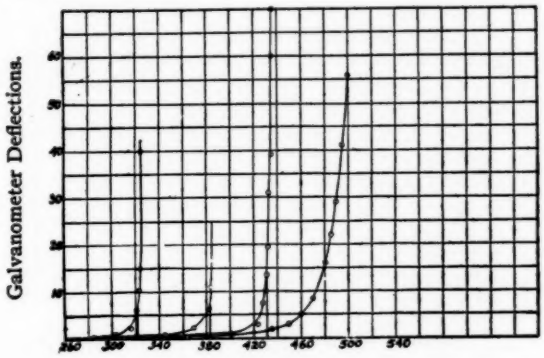
Fig. 5. The order of the illuminating voltages is not the same as that



Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Temp. 0° C. Gas pressure 3 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 0°$ C.

Fig. 7.

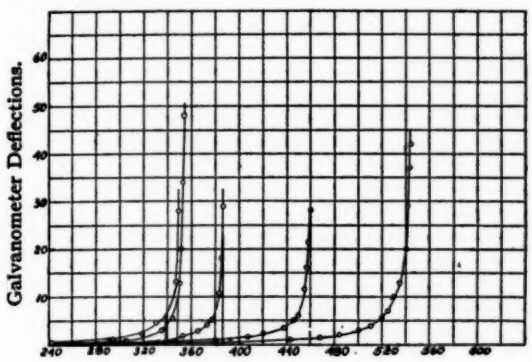
for values of D. The curve for $D = 0.5$ cm. lies between curves for $D = 1.0$ cm. and $D = 2.0$ cm. The curve $D = 1.0$ cm. shows best conditions for sensitiveness.



Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Temp. 38° C. Gas pressure 3 mm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 38°$ C.

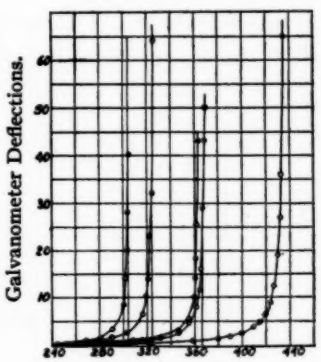
Fig. 8.

Fig. 6. The order of the illuminating voltages is regular. The conditions of sensitiveness are much better as represented in curve for $D = 2.0$ cm.



Variation of gas pressure reading from left to right 2–1–3–5–8 mm. Electrode distance 1 cm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f.

Fig. 9.

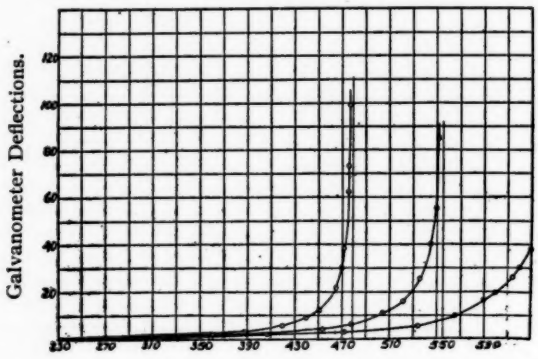


Variation of gas pressure reading from left to right 2–3–1–5–8 mm. Electrode distance 0.5 cm. Before forming. $A = 4.36$ cm². $L = 0.22$ C.f.

Fig. 10.

Fig. 7. The order of the illuminating voltages is regular. The best conditions of sensitiveness are represented by a curve that would lie between curves for $D = 0.5$ cm. and $D = 1.0$ cm.

Fig. 8. The order of the illuminating voltages is regular. The best conditions for sensitiveness are represented by a curve which will lie between curves for $D = 1.0$ cm. and $D = 2.0$ cm.

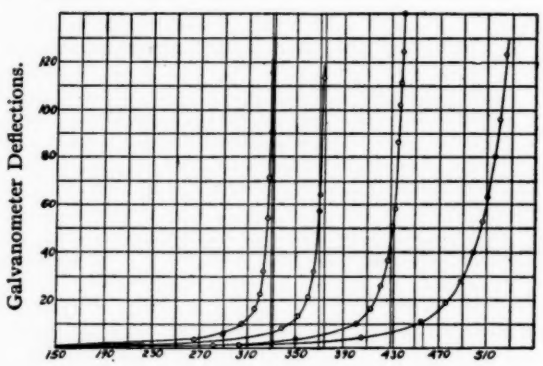


Voltage on Metal.

Variation of electrode distance reading from left to right 0.5–1–2 cm. Gas pressure 10 mm. After forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 24°$ C.

Fig. 11.

Fig. 9. The curve for $P = 1.0$ mm. lies between curves $P = 2.0$ mm. and $P = 3.0$ mm., showing that the critical pressure for minimum illuminating voltage is in the region of 2 mm. The best conditions of sensitiveness are represented by a curve which lies between curves for $P = 3.0$ mm. and $P = 5.0$ mm.



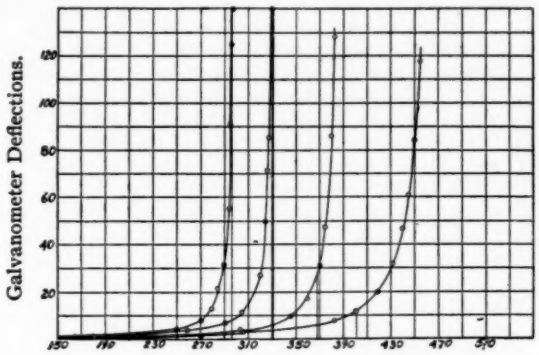
Voltage on Metal.

Variation of electrode distance reading from left to right. 0.5–1–2–3 cm. Gas pressure 3 mm. After forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 25°$ C.

Fig. 12.

Fig. 10. The curve for $P = 1.0$ mm. lies between curves for $P = 3.0$ mm. and $P = 5.0$ mm. This indicates that the critical pressure at

which the illuminating voltage is a minimum lies in the region of $P = 2.0$ mm. The best conditions for sensitiveness are represented by a curve which lies between curves for pressure between 2 and 3 mm.



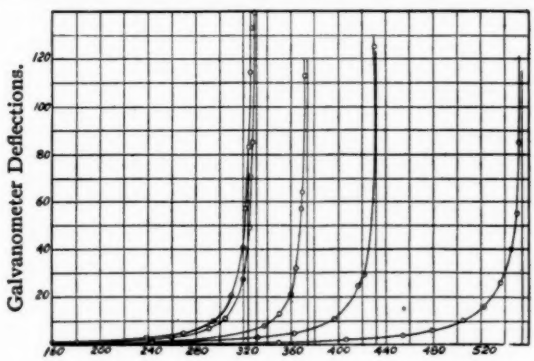
Voltage on Metal.

Variation of electrode distance reading from left to right 0.5–1–2–3 cm. Gas pressure 2 mm. After forming. $A = 4.36$ cm². $L = 0.22$ C.f. $t = 26°$ C.

Fig. 13.

The next set of figures, 11 to 15, represent values observed after the formation of the hydride. The following notes may be made.

Fig. 11. The order of illuminating voltages is regular. The best conditions for sensitiveness are shown by curve for $D = 0.5$ cm.



Voltage on Metal.

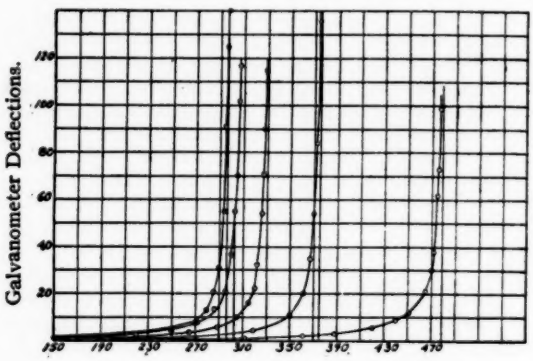
Variation of gas pressure reading from left to right 1–2–3–5–10 mm. Electrode distance 1 cm. After forming. $A = 4.36$ cm². $L = 0.22$ C.f.

Fig. 14.

Fig. 12. The order of illuminating voltages is regular. The best conditions for sensitiveness are shown by curve for $D = 0.5$ cm.

Fig. 13. The order of illuminating voltages is regular. The best conditions for sensitiveness are shown by a curve which lies between curves for $D = 0.5$ cm. and $D = 1.0$ cm.

Fig. 14. These curves show conditions of sensitiveness for different pressures and $D = 1.0$ cm. The curve representing best conditions for sensitiveness lies near the curve for $P = 3.0$ mm.

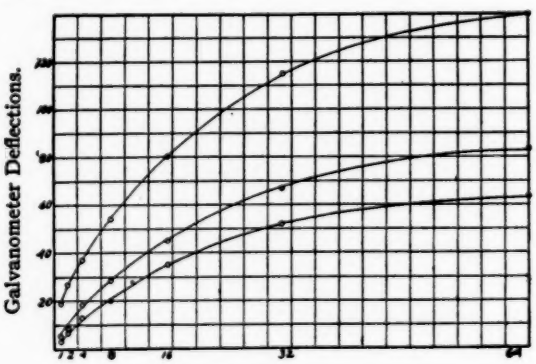


Voltage on Metal.

Variation of pressure reading from left to right 2–1–3–5–10 mm. Electrode distance 0.5 cm. After forming. $A = 4.36$ cm². $L = 0.22$ C.f.

Fig. 15.

Fig. 15. The curves show the conditions for sensitiveness for different pressures and $D = 0.5$ cm. The curve representing best conditions for sensitiveness lies near the curve for $P = 3.0$ mm.



Area of Metal Illuminated.

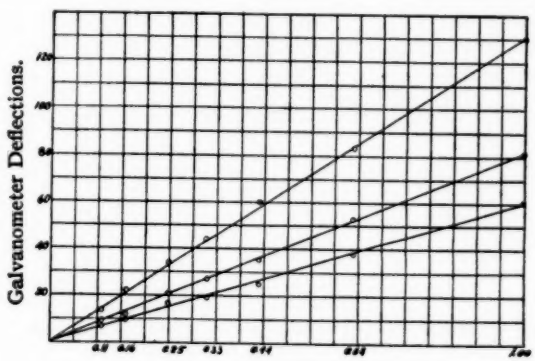
Variation of area of metal illuminated. Intensity of illumination 0.22 C.f. Gas pressure 2 mm. Electrode distance 0.5 cm. Voltage on metal reading from top downward 295, 294, 293 volts.

Fig. 16.

The curves of Fig. 16 show the variation of area of metal illuminated. The form of these curves indicates that variations in small areas illumi-

nated produce large changes in the current, and variations in large areas illuminated produce small changes in the current. These facts show that there is a maximum area of illumination for this particular type of photo-electric cell which, if exceeded, will not increase the sensitiveness.

The form of the curve should be a straight line if the electric field were uniform, the surface conditions uniform, with no reflection of the light, and no shadow caused by the electrode. In this cell the above conditions were not fulfilled, therefore the form of the curve is not a straight line. It follows finally a system of curves showing the variation of the intensity of illumination for three different voltages, the observa-



Variation of intensity of illumination. Electrode distance 0.5 cm. Voltage on metal reading from top downward 366, 365, 364 volts. After forming.

Fig. 17.

tions are taken after the forming of the cell. In Fig. 17 showing variation of current with intensity of illumination for $P = 5$ mm. the points for the smaller voltages lie more nearly on a straight line. The largest error in the intensity for the point farthest from the line is 7 per cent. of .15 C.f. or .01 C.f. The points for smaller intensities show much smaller deviations than those for larger intensities. The higher voltages applied caused unsteadiness of the current, hence, the galvanometer deflections are liable to larger errors.

BEST CONDITIONS FOR SENSITIVENESS.

By comparing the values for the critical voltages and currents obtained from the curves the best sensitive conditions may be selected.

A table of these values is given below for both before and after forming the hydride on the surface of the metal.

TABLE TO SHOW BEST CONDITIONS FOR SENSITIVENESS.

V = critical voltage.
I = critical current in galvanometer deflections.
D = best distance between electrodes in cms.
P = best pressure in mm.

Figure.	V	I	D	P
		Before Forming Metal.		
3	460 to 527	16 to 28	1 to 2	5.0
4	304	26	0.5	2.0
5	349 to 366	6 to 9	1 to 2	1.0
6	454	8	2	3[1]
7	335 to 396	10 to 45	0.5 to 1.0	3[2]
8	383 to 431	7 to 11	1 to 2	3[3]
9	387 to 460	28 to 29	1	3 to 5
10	304 to 324	26 to 32	0.5	2 to 3
		After Forming Metal.		
11	479	280	0.5	10
12	331	off scale	0.5	3
13	296 to 331	123 to 190	0.5 to 1.0	2
14	374	203	1	3
15	331	off scale	0.5	3

By inspection of the table above, it is seen that the best conditions for sensitiveness before forming are about

$$V = 300 \text{ volts.}$$
$$D = 0.5 \text{ cm.}$$
$$P = 2 \text{ to } 3 \text{ mm.}$$
$$t = 25° \text{ C.}$$

And the best conditions for sensitiveness after forming are about

$$V = 330 \text{ volts.}$$
$$D = 0.5 \text{ cm.}$$
$$P = 3 \text{ mm.}$$
$$t = 25° \text{ C.}$$

The cell is about 100 times more sensitive after forming than before forming.

THEORETICAL DEDUCTION OF MEASUREMENT OF INTENSITY OF ILLUMINATION.

If the intensity of illumination varies directly with the current for very small intensities, then it is possible to calculate the intensity measurable with an instrument of given sensibility.

[1] About − 20° C. salt and ice.
[2] 0° C.
[3] 38° C.

From Fig. 18 below it is seen that for the curve of 366 volts,

$$\tan\theta = \frac{d}{s} = \frac{130}{200} = 0.65,$$

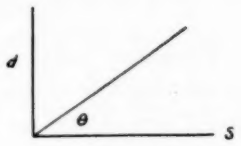


Fig. 18.

or

$$S = \frac{d}{\tan\theta} = \frac{d}{0.65},$$

$cd = I$ current flowing,

$c = 3.78 \times 10^{-10}$ amp. per mm. deflection,

$$d = \frac{I}{0.65 \times 3.78 \times 10^{-10}} \text{ mm.},$$

but

$$S = \frac{K}{l^2},$$

and for particular values of $S = 200$, and $l^2 = (100 \text{ cm.})^2$,

$$K = Sl^2 = 200 \times (100)^2 \text{ cm.} = 2 \times 10^6 \text{ cm.}$$

$$S = \frac{2 \times 10^6}{l^2}, \quad \text{or} \quad l = \sqrt{\frac{2 \times 10^6}{S}}.$$

Substitute value of

$$S = \frac{I}{0.65 \times 3.78 \times 10^{-10}}$$

in eq. above, then

$$l = \sqrt{\frac{2 \times 10^6 \times 0.65 \times 3.78 \times 10^{-10}}{I}} = \sqrt{\frac{4.9 \times 10^{-4}}{I}}.$$

By means of an electrometer a current $I = 10^{-12}$ can be measured. Substituting this value of I in the equation above, then,

$$l = \sqrt{\frac{4.9 \times 10^{-4}}{10^{-12}}} = 2.21 \times 10^4 \text{ cm. or } 221 \text{ meters},$$

the distance the 2.47 c.p. lamp could be removed from the cell and still be detected. By means of a tilted electroscope a current $I = 10^{-15}$ amperes can be measured.

Substitute this value in the equation,

$$l = \sqrt{\frac{4.9 \times 10^{-4}}{10^{-15}}} = \sqrt{4.9 \times 10^{11}} = 7 \times 10^5 \text{ cm. or 7 kilometers or 4.3 miles.}$$

The 2.47 c.p. lamp could be detected by means of an electroscope at a distance of 4.3 miles. To detect a candle instead of the 2.47 c.p. lamp at 4.3 miles distance by means of the tilted electroscope of 10^{-15} sensibility the distance could be as follows: Since the intensities of illumination of cell must be the same, then,

$$\frac{2.47}{(4.3)^2} = \frac{1}{l^2}, \quad l = \frac{4.3}{\sqrt{2.47}} = 2.7 \text{ miles.}$$

Professor Joel Stebbins,[1] of the University of Illinois, in his work on measuring the variation of intensity of illumination of the variable stars Algol and others used a selenium cell. It is possible to detect a candle at a distance of 500 meters, or 0.3 mile, with such a selenium cell.

The equation for the distance at which this potassium cell is sensitive when the current is measured with a tilted electroscope is

$$l_1 = \sqrt{\frac{K}{I_1}}$$

and for some other distance it is

$$l_2 = \sqrt{\frac{K}{I_2}},$$

$$\frac{l_1^2}{l_2^2} = \frac{I_2}{I_1}, \quad l_1 = 7 \times 10^5 \text{ cm.}, \quad l_2 = 5 \times 10^4 \text{ cm.},$$

then

$$\frac{l_1^2}{l_2^2} = \frac{49 \times 10^{10}}{25 \times 10^8} = 2 \times 10^2 \text{ approximately.}$$

Thus it is seen that the potassium cell is about two hundred times more sensitive than the selenium cell.

Energy Required to Produce an Ion.

To produce an ion a certain minimum amount of energy is required. This energy is that required to draw an electron out of an isolated molecule against the force of attraction of the positive charge of the molecule. Let R_1 be the radius of the molecule, e_1, the positive charge, e_2, the negative charge of electron. If the electron be displaced a distance dR the work done will be

$$dW = \frac{e_1 e_2}{R_1^2} dR.$$

The total energy required to draw an electron outside of the influence of the positive charge is

$$W = \int_{R_1}^{\infty} \frac{e_1 e_2}{R^2} dR = \frac{e_1 e_2}{R_1} \text{ units of work.}$$

$$e_1 = e_2 = 4.67 \times 10^{-10} \text{ E.S.U.}$$

$$R_1 = 10^{-8} \text{ cm. for hydrogen.}$$

(1) $$W = \frac{(4.67)^2 \times 10^{-20}}{10^{-8}} = 2.18 \times 10^{-11} \text{ ergs,}$$

[1] Astrophysical Journal, October, 1911.

the minimum amount of energy required to draw an electron from an isolated molecule of hydrogen or to produce a hydrogen ion.

For an isolated molecule it is necessary to withdraw the electron to an infinite distance from the center. If the molecule, however, is in an electric field the force of attraction between the molecule and the electron is zero at a distance say R_2 from the center. The effect of the electric field tends to decrease the energy required to withdraw the electron on one side of the molecule (with references to the direction of external electric field) while that on the opposite side is increased.

Let Fig. 19 represent the molecule of radius R_1; H is the direction of the external field in the direction of motion, Oa, of the electron. Let R_2 be the distance beyond which the electric attraction between the positive charge of the molecule and the electron is zero. The work required to withdraw the electron beyond the influence of the molecule is

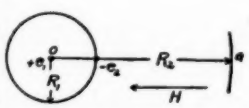


Fig. 19.

$$W = \int_{R_1}^{R_2} \frac{e_1 e_2}{R_2} dR = \frac{e_1 e_2}{R_1} - \frac{e_1 e_2}{R_2},$$

but $e_1 = e_2 = e$; and it is reasonable to assume $R_2 = 4R_1$.

$$W = e^2 \left(\frac{1}{R_1} - \frac{1}{4R_1} \right) = \frac{3}{4} \frac{e^2}{R_1}.$$

Substituting the values for R_1 and e,

$$(2) \qquad \frac{W = 3(4.67)^2 \times 10^{-20}}{4 \times 10^{-8}} = 1.63 \times 10^{-11} \text{ ergs.}$$

Thus the value of W in calculation (2) is three fourths that in calculation (1). Hence 2.18×10^{-11} ergs is not the minimum value.

The minimum amount of energy required to produce an ion by collision can be determined roughly from the data taken in this investigation. For the conditions of this work the minimum energy is $W = Eel$, where E is the potential gradient, or electric force, e is the charge of a negative ion or electron, and l is the mean free path of an ion. The force E can be determined roughly as follows: The voltage applied to the electrodes of the cell necessary to produce velocities high enough to cause ionization by collision, is obtained from the ionization curves. This voltage is shown very distinctly at the point on the curve where the ordinate or current shows an increase after the saturation state has been reached. Let V be this voltage taken from the curve, and let D be the distance between the electrodes. Then,

$$E = \frac{300D}{V} \text{ in E.S.U.}$$

(*a*) FIRST ASSUMPTION REGARDING MEAN FREE PATH OF ELECTRONS.

Assuming that the negative ions or electrons and the molecules of the hydrogen gas act as a mixture of two gases, the equation of the mean free path given by Maxwell in the kinetic theory of gases is

$$l_1 = \frac{1}{\pi N_1 \sigma_{11}^2 \sqrt{2} + \pi N_2 \sigma_{12}^2 \sqrt{1 + \frac{m_1}{m_2}}}$$

for the electrons,

$$l_2 = \frac{1}{\pi N_2 \sigma_{22}^2 \sqrt{2} + \pi N_1 \sigma_{12}^2 \sqrt{1 + \frac{m^2}{m_1}}}$$

for hydrogen molecules.

For an electron σ_{11} the diameter of the sphere of action when two electrons collide is practically zero. But σ_{12} the diameter of the sphere of action when an electron collides with a molecule of hydrogen is assumed equal to the radius of the molecule or 10^{-8} cm.

m_1 is the mass of an electron equal to 8.8×10^{-28} grams.

m_2 is the mass of a hydrogen molecule equal to 1.6×10^{-24} grams.

$$\frac{m_1}{m_2} = \frac{8.8 \times 10^{-28}}{1.6 \times 10^{-24}} = 5.5 \times 10^{-4} \text{ gram.}$$

This value is negligible in comparison with unity or $\sqrt{1 + \frac{m_1}{m_2}}$. The equation for the mean free path of the electron is

$$l_1 = \frac{1}{\pi N_2 \sigma_{12}^2} \text{ cm.}$$

In this equation N_2 is the number of molecules per cubic centimeter. From the kinetic theory of gases N_2 may be calculated. $PV = RT$, the gas law, which becomes $PV = \frac{2}{3}L = \frac{2}{3}N' = \frac{1}{2}mC^2$.

Where L is the average total kinetic energy.

N' is the number of molecules in volume V.

C^2 is the square of average velocities of molecules.

$\frac{1}{2}mC^2 = \alpha T$, where α is the universal constant.

$PV = \frac{2}{3}N'\alpha T$, or

$P = \frac{2}{3}N\alpha T$ for unit volume.

$N = \frac{3}{2} \cdot P/\alpha T$ the number of molecules per unit volume

$$= \frac{3}{2} \frac{13.6 \times 980h}{2 \times 10^{-16} T}; \; h \text{ is in cm.}$$

Suppose $T = 27 + 273 = 300$ for this work.

$$N = \frac{3}{2} \frac{13.6 \times 980}{2 \times 10^{-1} \times 300} h = 33 \times 10^{16} h \text{ in cm. mercury.}$$

Substitute this value for N_2 in the expression for the mean free path.

$$l_1 = \frac{1}{\pi \times 33 \times 10^{16} h \times 10^{-16}} = \frac{965 \times 10^{-5}}{h} \text{ cm.}$$

Applying the equation $W = Eel$,

$$e = 4.67 \times 10^{-10} \text{ E.S.U.}$$

$$E = \frac{V}{300D} \text{ E.S.U.}$$

$$l_1 = \frac{965 \times 10^{-5}}{h}.$$

For $h = 0.5$ cm., $D = 1.0$ cm., $V = 330$ volts,

$$W = \frac{4.67 \times 10^{-10} \times 965 \times 10^{-5}}{300 \times 1.0 \times 0.5} = 1.09 \times 10^{-11} \text{ ergs.}$$

An average of ten determinations gave 1.05×10^{-11} ergs for the minimum energy required to produce an ion in hydrogen by collision.

(*b*) SECOND ASSUMPTION REGARDING MEAN FREE PATH OF ELECTRONS.

If it be assumed that the negative ions or electrons occupy no space in the gas, or, that the hydrogen gas acts as though the electrons were not present, then the value of the mean free path is the same as that of the molecules. That is,

$$l_2 = \frac{1}{\pi N \sigma^2 \sqrt{2} h}$$

$$l_2 = \frac{685 \times 10^{-5}}{h} \text{ cm.}$$

For $h = 0.5$ cm., $D = 1.0$ cm., $V = 330$ volts,

$$W = \frac{4.67 \times 10^{-10} \times 685 \times 10^{-5} \times 330}{300 \times 0.5} = .70 \times 10^{-11} \text{ ergs.}$$

About 35 per cent. decrease from 1.05×10^{-11} ergs.

(*c*) THIRD ASSUMPTION REGARDING MEAN FREE PATH OF ELECTRONS.

By using the assumption that the mean free path of the electrons is $4\sqrt{2}$ times mean free path of the hydrogen molecules, as Bishop[1] did, the following results are obtained.

$$l_1 = 4\sqrt{2} l_2 = \frac{4\sqrt{2}}{\pi 33 \times 10^{-16} h \times 4 \times 10^{-16}} = \frac{1{,}360 \times 10^{-5}}{h},$$

$$W = \frac{4.67 \times 10^{-10} \times 1{,}360 \times 10^{-5}}{300} \times \frac{V}{Dh} = 21 \times 10^{-15} \frac{V}{Dh}.$$

[1] PHYSICAL REVIEW, 325, November, 1911.

For $P = 0.3$ cm., $D = 1.0$ cm., $V = 260$ volts,

$$W = 21 \times 10^{-15} \frac{260}{1 \times 0.3} = 1.82 \times 10^{-11} \text{ ergs.}$$

An average of 10 calculations gives 1.77×10^{-11} ergs.

To recapitulate, the values determined above and those by other investigators are:

1. For an isolated molecule the theoretical value is 2.18×10^{-11} ergs.

2. For a molecule in an electric field the theoretical value is 1.63×10^{-11} ergs.

3. The average of ten values determined from the data in accordance with the first assumption is 1.05×10^{-11} ergs.

4. The value determined in accordance with the second assumption is 0.70×10^{-11} ergs.

5. The average of ten values determined in accordance with the third assumption is 1.77×10^{-11} ergs.

6. Bishop obtained a value, by a method similar to that used in this work and in accordance with the third assumption used in the fifth determination, 1.58×10^{-11} ergs.

7. Rutherford[1] determined the energy required to produce an ion by the alpha particle. His value is 2.7×10^{-11} ergs.

8. Geiger,[2] and later Taylor,[3] using the same method, obtained about 5×10^{-11} ergs, and other investigators obtained values even as large as 10×10^{-11} ergs.

In the method, based on the ionizing power of alpha particles, it is assumed that their loss of kinetic energy is entirely transformed into energy of ionization, and that the decrease of the kinetic energy over a certain range divided by the total numbers of ions produced gives the energy required to produce one ion. Since a part of the kinetic energy of the alpha particle increases the average kinetic energy of the gas without producing ions, the ionizing energy is taken too large and the energy required to produce an ion is necessarily too large.

Value number (1) represents the minimum energy required to produce an ion when a molecule is isolated. This value is much larger than that for a molecule in an electric field. Rutherford's[4] determination is much nearer the value number (1) than any of the others.

Bishop's determination is very close to the value number (2), while my determination of the ionizing energy is slightly larger. Since the assumption regarding the mean free path in numbers 5 and 6 are the same, and

[1] Radio-Activity, second edition, p. 552.
[2] Proc. Royal Soc., Vol. 82, p. 486, 1909.
[3] Phil. Mag., p. 670, April, 1912.
[4] Radio-Activity, second edition, p. 552.

these values differ very slightly from the theoretical value number (2), it indicates that the assumptions made are not far from the truth.

Owing to lack of time and space an exact determination of the minimum energy required to produce an ion by collision is impossible; in the near future, however, it is hoped that this can be done with the data already in hand.

A design has been made for a sensitive photo-electric cell for photometric work in astronomy. It is expected to get a cell which will be sensitive enough to use instead of the erratic selenium cell now used.

SUMMARY AND CONCLUSIONS.

The following facts are established by this investigation for this type of photo-electric cell.

1. Owing to the low melting temperature of cæsium the use of this metal in photo-electric cells for photometric use is very impractical.

2. The temperature at which it is best to operate a potassium cell is about 25° C.

3. Cooling the potassium cell much below 25° C. does not increase its sensitiveness.

4. The sensibility of a potassium cell can be increased more than 100 times by the process of forming the hydride surface.

5. The distance between the electrodes for best sensitiveness is about 0.5 cm.

6. The hydrogen gas pressure at which the cell is most sensitive lies between 2 and 3 mm. of mercury.

7. The potential difference applied to the electrodes for most sensitive conditions is about 330 volts.

8. The minimum energy required to produce an ion by collision was calculated from the data and found to be of the order 1.77×10^{-11} ergs, while the theoretical value determined is 1.63×10^{-11} ergs.

9. Assuming that the straight lines obtained which show the relation between current and intensity of illumination hold for exceedingly small intensities, then by using a tilted electroscope of sensibility 10^{-15} amperes, a candle could be detected at a distance of 2.7 miles. This indicates that it is highly probable that a photo-electric cell could be used in astro-photometric work.

The author takes great pleasure in acknowledging his indebtedness to Professor A. P. Carman for the facilities for this investigation, and to Professor Jakob Kunz, both for his general supervision of the work and for many valuable suggestions.

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THE OPTICAL CONSTANTS OF SODIUM AND POTASSIUM.

BY R. W. AND R. C. DUNCAN.

INTRODUCTION.

DURING the early part of the nineteenth century it was found that light was partially polarized by reflection from the polished surface of a metal, and also that plane polarized light became, in general, elliptically polarized upon reflection from such a surface. Later this was shown to be due to a relative change in the phase of the two components. The amount of this change was first measured in 1847 by Jamin,[1] who was thus enabled to calculate, theoretically, the index of refraction of the metal used. The next important work along this line was done by P. Drude,[2] who, about 1890, completed a series of measurements from which he determined the optical constants of most of the common metals.

A large amount of work has since been done, so that at the present time the optical constants of almost all of the metals are known, with the exception of a few of the more highly oxidizable ones. On account of the rapid oxidation of these, the difficulty has been to obtain a bright metallic surface and to preserve it long enough to examine its optical properties. The few attempts that have been made, while not at all satisfactory, have shown that these metals offer an interesting field for further investigation. In 1898 Drude[3] made a single measurement with sodium light reflected from the molten surface of metallic sodium. This measurement gave an index of refraction of 0.0045 which, if correct, means that light travels 220 times as fast in that metal as in air. After making due allowance for some possible errors, which Drude admits might have lowered his result, he says that the index could not be greater than 0.054. But this value is still remarkably low compared to the indices of other metals.

Since Drude's value of the index of sodium was made with sodium light, the question arises is there any connection between the low value of the index and the fact that the light used was of a wave-length peculiar

[1] Ann. Chim. Phys. (3), Vol. 19, p. 296, 1847.
[2] Ann. d. Phys., Vol. 39, p. 481, 1890.
[3] Ann. d. Phys., Vol. 64, p. 159, 1898.

to that emitted by the sodium molecule. Might this be a phenomenon akin to resonance, or is the value of the index equally low for all colors? These questions, together with the extraordinarily low index obtained by Drude, suggested the investigation reported on the following pages.

THE PREPARATION OF THE SURFACE.

In his work with sodium, Drude obtained a bright metallic surface by melting the metal in an atmosphere of rarified hydrogen, but found it necessary to renew the surface every few minutes on account of its rapid oxidation. During one of these attempts to remove the thin layer of oxide the vessel containing the sodium was broken and he was unable to verify his first determination. In the present investigation a number of preliminary experiments were made to find, if possible, a method of obtaining a more permanent surface, free from oxide. As the metal can be kept indefinitely under oil, it seemed possible that a bright surface might be obtained and preserved by melting the metal under some light transparent oil. The attempts, however, were not satisfactory for, while a bright surface could be obtained in this way, it would remain bright but a few minutes. Then again on account of the large surface tension and small density of sodium, the molten surface assumed a shape almost spherical, so that a comparatively large surface would be required in order to obtain a small part of the surface approximately plane.

Some efforts were made to obtain the desired surface by melting the sodium in a vacuum, but here, too, it was difficult entirely to prevent oxidation. It was noticed, however, that bright surfaces were often obtained against the inside wall of the glass container. These surfaces remained bright after the sodium had solidified and could be preserved indefinitely by filling the vessel with dry air or oil. This suggested the possibility of obtaining a permanent sodium mirror against a plane glass plate. With this end in view a number of experiments were made. Many of these ended in complete failure, but others gave more promising results. Finally, by combining and improving several plans, a method was devised which has proved quite satisfactory.

The Method in Brief.—A little box, or capsule (Fig. 2), was made by cementing two plane glass plates to the ends of a short iron tube, or ring, from the side of which a small glass tube extended. The latter tube was about 5 cm. long and at its outer end was drawn out so as to leave a very small opening. This capsule was suspended in a cylindrical glass vessel, in the lower end of which had been placed some metallic sodium. The vessel was then closed and evacuated and the lower end immersed in an oil bath, which was heated to a temperature of about

110° C. Meanwhile the suspended capsule was being heated up to approximately the same temperature by an electric current passing through resistance grids held against the glass ends. When the sodium below had melted the capsule was lowered until the small end of the glass tube dipped into the molten metal. Then by admitting dry air into the containing vessel the liquid sodium was forced up through the glass tube into the capsule and pressed firmly against the plane glass ends, thus forming two bright sodium mirrors. After the whole apparatus had been allowed to cool, the sodium capsule was taken out, the small glass tube removed, and the opening closed with paraffin. In this convenient form the sodium mirrors could be kept indefinitely. The method was found to work equally well with potassium, and all sodium or potassium surfaces studied were obtained in this way.

Apparatus. — The containing vessel was made of glass and consisted of two parts (*A* and *B*, Fig. 1) fitting together at *a* in a ground junction. The lower part *A* was made by sealing the tube *D*, 5 cm. long and 1.5 cm. in diameter and closed at one end, into the larger tube *C*, 7 cm. long and 4 cm. in diameter. The upper part *B*, 9 cm. long and 4 cm. in diameter, was made from a glass tube closed at one end. At *f* was sealed in a side tube, which ended in the ground cone *h*. A glass rod *E* carried a cone *d* ground to fit *h*, and by means of the T-shaped end *H*, could be rotated about the line *cd* as an axis. It thus served as a windlass to raise and lower the capsule *F* which was suspended by a cord wrapped around the rod at *c*. The wires, carrying the current used to heat the capsule, were led in through the small tube *e*, which was closed with sealing wax. A rubber tube connected the outlet *k* to a two-way stopcock, one branch of which led to the air-pump, while the other, through drying tubes, led to the outside air. The vessel *AB* could thus either be evacuated through *k*, or when evacuated, could be refilled with dry air through the same tube.

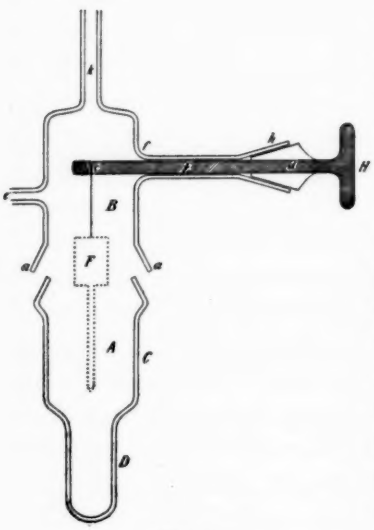

Fig. 1.

The short iron tube (*R*, Fig. 2) used in making the capsule, was 0.8 cm. in length and 1.5 cm. in diameter. A hole *a* 0.45 cm. in diameter was drilled radially through the side of the tube midway between the ends.

A fillet *s* 0.08 cm. wide and of the same depth was cut around the outside edge of each end, thus leaving the shoulder *t* on the inside edge. The end faces of these shoulders were made as plane as possible by grinding on fine emery paper backed by a surface plate. The two pieces of plane glass *gg*, after being carefully cleaned, were clamped tightly against these shoulders. The fillet along the outside edge of the metal ring, together with the adjacent glass plate, now formed a little groove which was filled with cement, while the shoulder pressing against the glass prevented the cement from creeping inside. The capsule was completed

Fig. 2.

Fig. 3.

by cementing the side tube *T* into the opening *a*. It was then baked at a temperature of about 120° C. for several hours, until the cement, which was at first a light yellow, became dark brown. In some cases a tube of brass (*S*, Fig. 3) was used for the side tube *T*. It was made from a brass rod, 0.45 cm. in diameter and 5 cm. long, which was tapered at one end almost to a point. A hole *o*, 0.3 cm. in diameter, was drilled along the axis of the rod to the point *n*. Then a very small hole 0.03 cm. in diameter was drilled from *m* to *n*, to connect with the larger opening.

Precautions to Prevent Oxidation.—As the most difficult problem in the whole research was the obtaining of a surface free from oxide, the precautions taken to prevent oxidation will be described in some detail.

In preparing to make a mirror, the containing vessel *AB* was first thoroughly cleaned. It was then placed in an upright position, and the capsule (*F*, Fig. 1) suspended from the rod *E* as already described. The vessel was alternately evacuated and filled with dry air[1] a number of times, in order to remove any water vapor or other gases from the interior. Finally it was left closed, full of dry air. The vessel was further protected from moisture by a U-tube of phosphorus pentoxide, placed between it and the pump.[2]

[1] The air was dried by drawing it through U-tubes of sulphuric acid and phosphorus pentoxide.

[2] Gaede's combination of oil and mercury pumps was used.

The quantity of sodium, a part of which was to be used in making the mirror, was immersed in a light paraffin oil. By means of a cork borer of a diameter just smaller than that of the tube (*D*, Fig. 1), a cylindrical stick about 3 cm. long was cut out of the metal. Any heavy coating of oxide still adhering to the ends of the stick was removed with a knife. The cylinder of sodium, thus obtained, was quite free from oxide, since all the cutting had been done under oil. The stick was lifted out and rinsed thoroughly in benzine to remove the oil, the benzine, in turn, being removed by rinsing in toluene. Now, as quickly as possible, the glass vessel *AB* was opened, the sodium cylinder dropped into the receptacle *D*, and the vessel immediately closed and evacuated. The remaining toluene was therefore rapidly removed by evaporation at a reduced pressure. It will be noted that at no time during the whole process was the sodium exposed to the open air. In spite of all these precautions, however, a surface film of oxide would always form before the sodium was entirely melted. Therefore, in order to pierce this surface film, and to permit as little as possible of it from entering the capsule, it was necessary to make the tube (*T*, Fig. 2) pointed, with a very small opening in the end (as already described).

It was possible in this way to obtain bright surfaces, but, upon a close examination, they proved to be covered with an extremely thin filmy veil. This film, however, was not spread uniformly over the surface, but was distributed in curved and ring-like figures, leaving in a number of cases, small areas which were entirely free from it. Further experiment proved that this thin film was due to a spreading out of a small quantity of oxide which had entered the tube *T* when it broke through the surface of the molten metal.

In order to prevent this, two methods were used with about equal

Fig. 4.

Fig. 5.

success. In the first method a wedge-shaped device was used to cut the surface film and open a way through it for the tube *T*. This device (Fig. 4) was made by fastening two very thin flat steel springs *pp*, each 1.3 cm. long and 0.5 cm. wide, to the metal disk *d* so that the outer ends of the springs were held together, thus making a sort of hollow wedge as shown in the figure. A hole, 0.6 cm. in diameter, was drilled through

the center of the disk, which was 1.2 cm. in diameter, just smaller than the inside diameter of D (Fig. 1). This was suspended from the capsule so that the tube T extended through the hole in the disk, with the pointed end inside the hollow wedge. Now as the capsule was lowered, the wedge pierced the oxide film and entered the sodium, stopping when the disk rested upon the surface of the metal. By a further lowering of the capsule, the sides of the wedge were forced apart by the pointed tube. This pushed aside the surface film so that the point of the tube could enter directly the bright surface of the metal.

In the second method a glass tube (W, Fig. 5), whose outside diameter was slightly less than the inside diameter of the receptacle D, was drawn down at one end to an opening h about 0.8 cm. in diameter. This tube was then suspended, small end downward, from the capsule (F, Fig. 1), so that the lower end of the tube W was about 1 cm. below the point m of the small tube T. When, upon lowering this contrivance, the lower end of W reached the metal some of the surface film would, of course, enter at h. As the tube W continued to descend, the liquid metal, entering through h from below, burst through and pushed aside this film (which adhered to the sides of the tube W), thus permitting the point of the tube T to enter a bright globule of melted sodium.

Both of these devices were of considerable help in obtaining bright surfaces, but neither could be said to be consistently successful. By making a number of mirrors, however, a few were obtained which were practically free from this surface film. The results given in this report were obtained from these surfaces.

Other Surface Defects.—In some cases the surface of the mirror was found to be marred by numerous fine lines along which the sodium seemed to have drawn away from the glass. These lines suggested a crystalline formation and probably had been caused by the contraction of the sodium upon solidifying. This configuration on the surface was especially noticeable when the mirror was examined with polarized light, for, since the light totally reflected from the back surface of the glass along these lines differed in phase from that reflected from the sodium surface, both could not be extinguished at the same time. Fortunately, however, only a few of the mirrors showed this defect, and they were either entirely discarded or measurements made only on that part of the surface from which these lines were entirely absent.

A number of failures proved that unless the glass plates, upon which the mirrors were to be formed, were heated to approximately the temperature of the melted sodium, the metal would solidify as soon as it touched the plates, leaving streaks or striations across the surface.

It was also found necessary to heat the tube T, through which the sodium was forced up into the capsule, for, otherwise the sodium would solidify while yet in the tube, and so never reach the glass plates. On this account a brass tube proved much more convenient than the glass one originally used, since the whole of the brass tube could be heated by merely heating the capsule.

Cement.—In making the capsule quite a little difficulty was experienced in finding a satisfactory cement, *i. e.*, one which would fasten together glass and iron in an air-tight junction and would not soften or become porous when heated in a vacuum to the required temperature (about 110° C.). A number of cements were tried, some being commercial products, while others were mixed in the laboratory according to various formulæ. The one finally selected is on the market under the trade name "Rock Cement." When first tried, this was used according to the printed directions, but was not satisfactory. Further experiment, however, showed that it could be made so if thoroughly dried by heating. It was also noticed that better results were obtained by allowing the cement to thicken somewhat by evaporation before using.

MEASUREMENTS.

Method and Apparatus.—Drude's method for obtaining the optical constants of metals was used throughout this investigation. It consists in an examination of polarized light reflected from the polished surface of the metal. If the incident light be polarized in a plane making an angle of 45° with the plane of incidence, the quantities to be measured are ϕ, the angle of incidence; Δ, the phase change introduced by reflection; and ψ, the azimuth of restored polarization. The optical constants (n, the index of refraction and κ, the index of absorption) are obtained by substitution in the following formulæ, given by Drude.[1]

$$n^2(1 - \kappa^2) = S^2 \cos 2Q + \sin^2 \phi, \tag{1}$$

$$2n^2\kappa = S^2 \sin 2Q, \tag{2}$$

where

$$\cos 2P = \sin 2\psi \cos \Delta, \tag{3}$$

$$\tan Q = \tan 2\psi \sin \Delta, \tag{4}$$

$$S = \sin \phi \tan \phi \tan P. \tag{5}$$

Writing

$$S^2 \cos 2Q + \sin \phi = A, \tag{6}$$

$$S^2 \sin 2Q = B, \tag{7}$$

then

$$2n^2 = \sqrt{A^2 + B^2} + A, \tag{8}$$

$$\kappa = \frac{\sqrt{A^2 + B^2} - A}{B}. \tag{9}$$

[1] Ann. d. Phys., Vol. 64, pp. 161–2, 1898.

The principle angle of incidence $\bar{\phi}$, the principal azimuth $\bar{\psi}$, and the reflecting power R, are given by:

$$\sin^4 \bar{\phi} \tan^4 \bar{\phi} = n^4(1 + \kappa^2)^2 - 2n^2(1 - \kappa^2) \sin^2 \bar{\phi} + \sin^4 \bar{\phi}, \quad (10)$$

$$\bar{\psi} = \tfrac{1}{2} \tan^{-1} \kappa, \quad (11)$$

$$R = \frac{n^2(1 + \kappa^2) + 1 - 2n}{n^2(1 + \kappa^2) + 1 + 2n}. \quad (12)$$

Since for sodium and potassium the constant A (Eq. 6) is negative, the calculations may be much simplified. Making A negative and expanding the radical, equation (8) becomes

$$2n^2 = A\left(\frac{B^2}{2A^2} - \frac{B^4}{8A^4} + \text{etc.}\right).$$

As B is small compared to A, all terms except the first may be neglected. This gives

$$n = \frac{B}{2\sqrt{A}}. \quad (13)$$

In a similar way, equation (9) may be simplified, giving

$$\kappa = \frac{2A}{B}. \quad (14)$$

In all calculations made in this report an error of less than one per cent. results from using equations (13) and (14).

As in Drude's experiments, the instrument used was a spectrometer fitted with a polarizing and an analyzing nicol and a Soleil-Babinet compensator. The desired wave-length was obtained by means of a monochromatic illuminator, the source of light being an electric arc. For each particular wave-length used, the zero position and the constant of the compensator were accurately determined.

Since the formula for n involves the tangent of the angle 2ψ, usually large, a small error in measuring this angle has a much greater effect upon the result than a corresponding variation in Δ. This is especially true of the light reflected from a sodium surface, for which 2ψ is about 89°.

A number of careful measurements, made to determine the most accurate method of setting the nicol, showed that the best results were obtained by removing the eyepiece of the telescope and looking directly at the nicol. Viewed in this way, there appeared, at the extinction point, a broad and quite black horizontal band which never entirely covered the field.[1] Upon sliding the wedge in the compensator this band would

[1] This is not the phenomenon so often noticed with crossed nicols, for the band appeared only when the compensator was in position. With the new compensator (mentioned elsewhere in this article) the field was uniform, but if the quartz plates were rotated slightly upon each other, the band appeared and became quite distinct.

move up or down in the field, while upon slightly moving the nicol in either direction from an extinction point, it would slowly fade away, usually moving across the field in the direction of its length. The cause of this unexpected appearance of the field has been traced to a fault in the construction of the compensator, the principal axes of the quartz plates not being exactly perpendicular to each other.

The extinction point for both the compensator and the nicol was taken to be that position for which the dark band was centered in the field. For the compensator, the value used for the extinction point was the mean of eight or ten settings. For the nicol, a larger number of readings was necessary, since, under the best conditions, the individual settings varied by approximately thirty minutes of arc. Therefore, in order to get readings in every possible position of both the polarizer and the analyzer the following method was adopted. The polarizer was set in four positions, each 45° from the plane of incidence. At each of these positions, seven to ten readings of the analyzer were taken for each of its two extinction points, 180° apart. Therefore the value obtained for 2ψ was the mean of 50 to 80 readings.[1]

Mounting the Mirrors.—The mirrors were mounted so as to be viewed through a prism. A drop or two of cedar oil, which had approximately the same index of refraction as the glass, was placed upon the surface of the mirror and the mirror pressed firmly against the hypotenuse side of a right-angled prism. The light entered one leg of the prism normally and, after reflection at the sodium surface, emerged normally through the other leg. By this plan all the troublesome reflections from the front surface of the glass of the mirror were avoided, and, since the light was normal to both prism faces, it suffered no change of phase or of azimuth, either at the point of incidence or emergence. In order to remove any film of grease or other foreign matter, which might cause a phase change, the mirror and all of the surfaces of the prism were carefully washed before mounting.

Tests for Possible Corrections.—Should there be a slight difference between the value of the refractive index of the cedar oil and that of the glass, a change in the azimuth of polarization would be produced at the boundary between the two media. A test for such an effect was made in two ways, as follows:

First, a piece of plate glass was fastened with the cedar oil to the hypotenuse side of the prism. Polarized light was sent through the prism in the same way as before, but was now totally reflected from the

[1] A new compensator combined with a half shadow analyzer (Zehnder, Ann. d. Phys., Vol. 26, p. 985, 1908) has since been received from Germany, and the few measurements taken with it agree closely with those reported in this paper.

back surface of the glass plate. A careful examination of this light which, it will be noticed, had passed twice through the oil film, each time at an angle of 45°, showed no change in the azimuth of polarization. The phase change was also measured and found to agree closely with that to be expected from total reflection.

Secondly, five square pieces of glass, cut from the plate used in making the sodium mirrors, were placed between 60° prisms as shown in Fig. 6. A drop of cedar oil was placed between each plate, and between the first and last plates and the prism faces, and the whole pressed firmly together. Polarized light was sent through the combination in such a way as to pass normally through the outer prism faces (as indicated in the figure). The light therefore traversed twelve surfaces of contact between glass and oil, the angle at each being 60°. Here again a careful set of measurements failed to show any change in the azimuth of polarization. Hence, any effect, if present, must be negligible.

Fig. 6.

Another possible source of error was the presence of the oxide film, of which, as already mentioned, almost every sodium mirror showed at least a trace. In order to determine the optical effect of this film, the phase change and restored azimuth were measured on six different sodium mirrors. A week later a second set of measurements were made on the same surfaces. In the results, which are given below (Table I.), the phase change is expressed in head divisions of the micrometer screw and may be reduced to degrees by multiplying by 0.3937. The numbers given in the first column are used merely to distinguish the individual surfaces.

TABLE I.

Mirror No.	Phase Change.			Azimuth.		
	1st.	2d.	Av.	1st.	2d.	Av.
1	329	327	328	89° 21′	89° 23′	89° 22′
2	322	334	328	89° 20′	89° 24′	89° 22′
3	310	306	308	89° 6′	89° 13′	89° 10′
4	320	305	312	89° 3′	88° 54′	88° 58′
5	330	329	329	88° 54′	89° 3′	88° 58′
6	316	317	317	88° 50′	89° 0′	88° 55′

The results have been arranged in the above table in descending order of the average azimuth, and an examination of the individual surfaces showed that in general the film became more pronounced in the same order. No such variation of phase change could be observed. A com-

parative study of the first and second columns in both the phase change and azimuth above indicates no consistent change in the surfaces during the week which elapsed between the two sets of readings.[1]

As Nos. 1 and 2 were practically free from film, they were chosen for the determination of the optical constants of the metal as given below. It is believed that the results obtained from these mirrors are very near to those that would be obtained from a perfect surface, since the rather heavy film on Nos. 5 and 6 reduced the azimuth by less than thirty minutes.

Nos. 1, 3 and 4 were made from (Merck) sodium, guaranteed to be free from other metals, while Nos. 2, 5 and 6 were made from ordinary commercial sodium. The results did not show any consistent difference between the two specimens of the metal.

It is interesting to note that Nos. 5 and 6 were mirrors on the opposite sides of the same capsule. No. 5 was made directly on the hypotenuse face of a prism (by substituting a prism for one of the glass plates of the capsule), and so could be examined without the use of the oil film, while No. 6 was formed on the glass plate and examined through the oil. The results indicate that the oil film had no appreciable effect.

Method Applied to Mercury.—Exactly the same method was used throughout to determine the optical constants of mercury. As the constants for this metal are well known the results (Table II.) serve as a check upon the method. It will be noticed that the values for n are slightly higher than those obtained by Drude[2] or by Meier.[3] This may be due to the particular specimen of mercury used.

TABLE II.

Mercury. $\phi = 45°$.

λ	Δ	2ψ	n	κ
6,650	162° 11′	82° 57′	2.34	2.47
5,893	160° 4′	83° 1′	1.92	2.78
4,720	156° 14′	83° 18′	1.35	3.42

Former values obtained are:

Drude, $\lambda = 5{,}893$, $n = 1.73$, $\kappa = 2.87$.
Meier, $\lambda = 5{,}893$, $n = 1.62$, $\kappa = 2.71$.

Specimen Set of Readings in Detail.—In order to give an idea of the accuracy of setting and the method of procedure, a complete series of readings, taken to determine a single value of n and κ, is given in Table III. Every result was determined from a similar series of readings.

[1] Measurements taken on some of these surfaces three months later showed no change in the character of the surface.

[2] Ann. d. Phys., Vol. 39, p. 530, 1890.

[3] Ann. d. Phys., Vol. 31, p. 1031, 1910.

TABLE III.

A. Determination of Zero Point and Constant of Compensator.

Sodium mirror No. 1. $\phi = 44° 59'$.

	For Phase Difference of		
	-2π	Zero.	$+2\pi$
	(0) 80.0	(12) 37.0	(23) 91.0
	81.0	36.0	95.0
	79.2	36.0	98.0
	81.0	38.5	99.0
	78.3	38.0	(24) 1.0
	81.0	38.0	(23) 95.0
	78.2	36.5	96.2
Averages.....	(0) 79.8	(12) 37.0	(23) 96.3

Numbers in ( ) indicate whole number of turns of the micrometer screw. The readings are in head divisions.

2,316.5 divisions = 720° (4π). Therefore 1 div. = 720/2,316.5 deg.

B. Determination of Phase Change upon Reflection.

Readings for Extinction	
At Beginning.	At End of Series.
(16) 90.0	(16) 81.5
89.0	83.0
88.0	84.2
86.0	84.0
86.5	82.5
87.2	85.0
88.6	86.0
Av. (16) 87.9	(16) 83.7

1,685.8 − 1,237 = 448.8 divisions.

Therefore $\Delta = 139° 29'$.

General average, 1,685.8.

C. Determination of Restored Azimuth.

Polarizer.	49° 39′		229° 39′	
Analyzer.....	360° 9′	179° 29′	179° 48′	359° 42′
	359° 22′	179° 17′	180° 7′	359° 40′
	359° 30′	180° 2′	180° 31′	359° 42′
	360° 20′	179° 4′	180° 32′	359° 4′
	360° 8′	179° 33′	179° 46′	359° 40′
	359° 59′	179° 23′	179° 44′	359° 32′
	360° 11′	179° 52′	179° 40′	359° 46′

General average (reduced to one position), 359° 46.2′.

Polarizer.	139° 39′		319° 39′	
Analyzer.....	270° 40′	90° 53′	90° 27′	270° 1′
	270° 24′	90° 9′	90° 22′	270° 14′
	270° 22′	90° 18′	90° 28′	270° 53′
	270° 1′	90° 32′	89° 48′	270° 11′
	270° 24′	90° 28′	90° 28′	270° 31′
	270° 11′	90° 9′	89° 53′	270° 10′
	270° 42′	90° 19′	90° 2′	270° 54′

General average, 270° 20.9′. Therefore $2\psi = 89° 25.3′$.
By substitution $A = -3.1553$, $B = 0.1135$. Giving $n = 0.032$, $\kappa = 55.6$.
Index of refraction for glass plate = 1.5 (approximately).[1]
Therefore the index of refraction from air to sodium is:

$$n = 1.5 \times 0.032 = 0.048.$$

RESULTS.

TABLE IV.

Sodium.

Sodium mirror No. 1. $\phi = 44° 59′$.

λ	Δ	2ψ	n	κ
6,650	139° 29′	89° 25.3′	0.048	55.6
5,893	134° 46′	89° 22.9′	0.042	54.3
5,460	130° 34′	89° 14.1′	0.045	44.7
4,720	124° 34′	89° 0′	0.051	34.2
4,350	117° 58′	88° 46.7′	0.053	26.4

Sodium mirror No. 2. $\phi = 44° 59′$.

λ	Δ	2ψ	n	κ
6,650	142° 1′	89° 25.9′	0.053	54.5
5,893	137° 10′	89° 24.7′	0.045	55.8
5,460	136° 6′	89° 9.8′	0.060	40.6
4,720	130° 20′	88° 57.5′	0.062	32.4
4,350	127° 7′	88° 32.0′	0.063	37.0

The averages from the above are given in the second and third columns below. The values of $\bar{\phi}$, $\bar{\psi}$, and R, given in the last three columns, are obtained by using these averages in the formulæ (10), (11), and (12).

λ	n	κ	$\bar{\phi}$	$\bar{\psi}$	R
6,650	0.051	55.0	72° 11′	44° 29′	97.7
5,893	0.044	55.0	68° 51′	44° 29′	97.1
5,460	0.052	42.6	68° 48′	44° 20′	96.5
4,720	0.057	33.3	66° 29′	44° 9′	95.2
4,350	0.058	31.7	66° 0′	44° 6′	94.8

TABLE V.

Potassium.

Potassium mirror No. 1. $\phi = 44° 59′$.

λ	Δ	2ψ	n	κ
6,650	123° 23′	88° 49.7′	0.057	28.2
5,893	116° 26′	88° 36.7′	0.059	22.7
4,720	106° 18′	88° 27.6′	0.060	15.5

[1] Any error in this approximation is within the limits of experimental error.

Potassium mirror No. 2. $\phi = 44° 59'$.

λ	Δ	2ψ	n	κ
6,650	128° 29′	88° 39.0′	0.075	25.4
5,893	123° 19′	88° 25.5′	0.077	21.6
4,720	114° 19′	88° 5.2′	0.079	13.1

Averages, etc.

λ	n	κ	$\overline{\phi}$	$\overline{\psi}$	R
6,650	0.066	26.8	65° 27′	43° 56′	93.8
5,893	0.068	22.1	62° 58′	43° 42′	92.0
4,720	0.070	14.3	57° 9′	43° 0′	86.9

Although the two potassium mirrors studied seemed to be entirely free from the oxide film, the results are not considered conclusive until more surfaces have been examined. No explanation is offered, at present, for the difference in the results obtained from the two specimens.

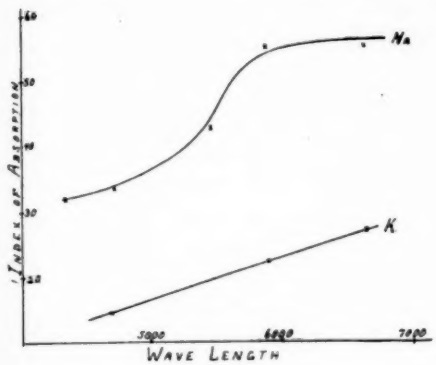

Fig. 7.

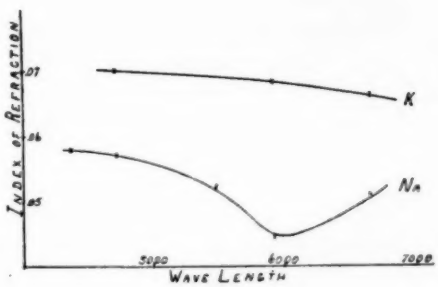

Fig. 8.

The variations of the optical constants with wave-length are shown in Figs. 7 and 8. The curve for the index of refraction of sodium has a distinct minimum near the D-line.

Conclusions.

1. It is possible to obtain and preserve indefinitely bright surfaces of both sodium and potassium.

2. Metallic sodium has the lowest index of refraction and the highest reflecting power of any metal known. This is in agreement with Drude's observations. It is interesting to note that the value for the refractive index for sodium light is very near (slightly less than) the upper limit set by Drude.

3. While the refractive index for sodium is very low for all wavelengths, it apparently has a minimum close to the sodium line.

4. Next to sodium, potassium has the lowest index of any metal. Its reflecting power, however, is slightly less than that of either silver or sodium.

5. The method seems applicable to the sodium-potassium alloy, and possibly may be adapted to other highly oxidizable substances. By using quartz, or uviol glass, plates and prisms the investigation may be carried into the ultra-violet by means of Minor's method.[1]

While the results reported above seem to be fairly consistent among themselves, the authors intend to verify them by further investigation. They hope, also, to be able to determine the optical properties of the sodium-potassium alloys, and probably of some other substances, to which the same method is applicable.

In conclusion we wish to acknowledge our indebtedness to Professor Arthur W. Goodspeed, who placed at our disposal all the facilities of the laboratory and kindly procured for us additional apparatus. We wish also to thank Professor Horace C. Richards for suggesting the subject of this research and for his continued interest and coöperation.

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May, 1912.

[1] Ann. d. Phys., Vol. 10, p. 581, 1903.

A METHOD OF PRODUCING KNOWN RELATIVE SOUND INTENSITIES AND A TEST OF THE RAYLEIGH DISK.

BY G. W. STEWART AND HAROLD STILES.

THE absolute intensity of sound has been measured principally in four ways, viz., by the use of the Rayleigh disk placed directly in the sound,[1] by the measurement of the increased pressure at a reflecting wall,[2] by measuring pressure changes at nodes of stationary waves by a manometer,[3] and by optical interference methods.[4] In some of the experiments to which reference has just been made, varying measurable sound intensities have been produced, but in such a manner as to be unavailable for the calibration of intensity measuring devices of various kinds. Indeed, we have found no record of a successful effort to produce known varying intensities available for testing purposes. The application of the inverse square law is quite inaccurate, even out of doors. The construction of a sound-proof or a "silence" room will probably not reduce the reflection sufficiently to justify the assumption of the variation of the intensity inversely as the square of the distance.

The theory[5] of the acoustic shadow produced at any distance from a rigid sphere with the source located on the sphere suggested a method of producing known variations of intensity and thus obtaining a calibration device for sound-measuring apparatus.

The theory can be briefly stated. Let the source be confined to a small area on the surface of the sphere within which $P_n(\mu) = 1$. Let the velocity of this source region be simple harmonic and let it have the same magnitude U throughout. The following notation and equations are assumed:

ψ represents the velocity potential,
a represents the velocity of sound,
r represents the distance from center of sphere,
c represents the radius of the sphere,

[1] Zernov, Annal. d. Phys., 26, 1908, p. 79, Fig. 10.
[2] Altberg, Annal. d. Phys., 11, 1903, p. 405, and Zernov, Annal. d. Phys., 21, 1906, p. 131.
[3] Raps, Annal. d. Phys., 36, 1889, p. 273.
[4] Raps, Annal. d. Phys., 50, 1893, p. 193, and Sharpe, Science, 9, 1910, 1909, p. 808.
[5] Stewart, PHYS. REV., Vol. ~~XXXVIII.~~, No. 6, December, 1911.

dS represents an element of surface,

$$K = \frac{2\pi}{\text{wave length}},$$

$$\gamma = k(at - r + c),$$

$$F = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\alpha' + \beta\beta'}{\alpha^2 + \beta^2},$$

$$G = \sum \frac{2n+1}{2} P_n(\mu) \frac{\alpha\beta' - \alpha'\beta}{\alpha^2 + \beta^2},$$

$$^2f_n(ikr) = \alpha' + i\beta',$$

$$^2F_n(ikc) = \alpha + i\beta.$$

Then

$$\dot{\psi} = \frac{ka}{2\pi r}(F \sin \gamma + G \cos \gamma) \iint U dS. \qquad (1)$$

The energy per unit volume is $\frac{1}{2}\rho_0 a^2 s^2$, where ρ_0 is the density and s the condensation. But s equals $-\dot{\psi}/a^2$. Therefore we have for the mean potential energy the following expression,

$$\text{Energy per unit volume} = \tfrac{1}{2}\rho_0 \frac{\dot{\psi}^2}{a^2} = \tfrac{1}{2}\rho_0 (F^2 + G^2) \left(\frac{k}{2\pi r} \iint U dS\right)^2. \qquad (2)$$

We determined to construct a sphere with a satisfactory source of sound thereupon, and thus to secure in the region of the sphere varying sound intensities of known relative values.

As shown in Fig. 1 the sphere was mounted on the edge of the roof of the Physics Building. It was placed on the side where the building was 21 meters high. There were neither buildings nor trees within several hundred feet, and this, combined with the high elevation, made the location very satisfactory. Indeed, the only reflecting surface was the roof. The arrangement of the apparatus shows that the error due to the reflection from the roof would be very small, and this error was further reduced by a covering of three fourths of an inch of hair felt.

The sphere was constructed of cement, the wall thickness being 5 cm. The diameter of the opening, the source of sound, was about 5 cm. The circumference of the sphere was 135.9 cm. As shown by the figure, the sphere could be rotated readily, the angle being indicated at the water seal. The sphere was supported by a horizontal 5 cm. pipe 230 cm. above it, and this pipe was in turn supported by two 5 cm. pipes and an iron flagstaff, all three being distant from the sphere at least 250 cm. The reflection from these supports was practically nil.

[1] $f_n(ikr)$ and $F_n(ikc)$ are defined in Rayleigh's Theory of Sound, Vol. II., and in article by Stewart, loc. cit.

The sound was produced by an electromagnetically operated tuning fork mounted on a resonator, the latter being introduced into a funnel located at a distance of 700 cm. from the sphere and connected to it through an iron pipe as suggested by Fig. 1. The frequency used was 256, and thus *kc* was very approximately unity.

The apparatus to the left of the sphere in Fig. 1 is a Rayleigh disk

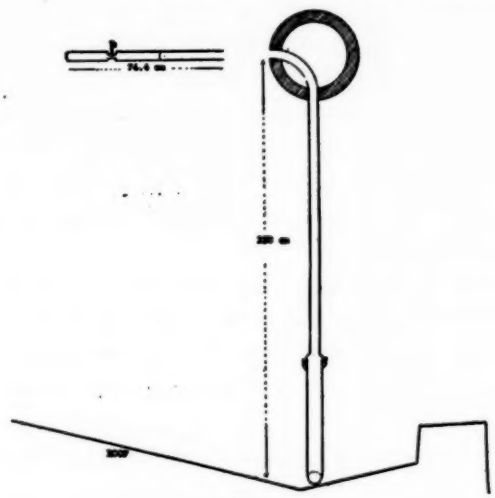

Fig. 1.

device. This was used to prove the practicability of this method of producing known relative intensities. This device is a modification of the one suggested by Rayleigh[1] and is drawn to scale in Fig. 2. It was made of brass tubing. The constriction in the tube was introduced to increase the sensitiveness. The dimensions needed were calculated by an approximate formula and then tested experimentally before constructing the apparatus.

The mirror, 0.6 cm. in diameter, was made from a very thin microscope

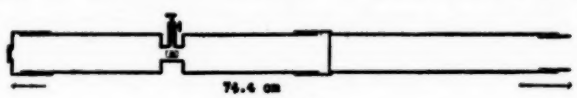

Fig. 2.

cover glass. It was suspended by a quartz fiber. The complete period was 6 seconds.

The observing telescope was placed along the axis of the tube and the scale parallel to the tube and in front of the mirror window. The apparatus had a high sensibility giving a definite deflection for what would be

[1] Rayleigh, Phil. Mag., Vol. XIV., p. 186, 1882.

termed a "faint" sound. The sensitiveness could have been increased greatly by reducing the size of the fiber suspension, but for work out-of-doors a short period was highly desirable.

The diaphragm was a piece of thin letter paper, but the selection of this material has no significance as it was not the result of investigation. The Rayleigh disk as used undoubtedly gives a deflection which is practically proportional to the kinetic energy on the interior at the disk itself. Indeed this has been demonstrated[1] experimentally. So far as we have been able to ascertain there is no experimental evidence that the kinetic energy at the disk itself is proportional to the potential energy which would exist at the opening of the resonating tube if the presence of the apparatus produced no distortion. Yet we here tentatively assume this to be the case. In this paper "energy" refers to potential energy unless otherwise designated.

We used the disk at three distances, viz., $kr = 2$, $kr = 3$, and $kr = 4$. The computations for these distances were made in accord with the formula (2) and the relative values of $F^2 + G^2$ obtained. The values for $f_n(ikr)$ and $F_n(ikc)$ were computed from equations defining these expressions.[2]

The values of the terms of Legendre's series $P_n(\mu)$ for the angles used from 0° to 180° were ascertained from tables and the relations

$$P_{2n+1}(90° + \theta) = P_{2n+1}(90° - \theta)$$

and

$$P_{2n}(90° + \theta) = P_{2n}(90° - \theta).$$

The terms were retained as far as $P_6(\mu)$. The computations are probably sufficiently accurate for the purposes of this paper. The results are shown in the accompanying tables.

The accompanying curves (Fig. 3 and Fig. 4) show these computations plotted with the value at 0° taken as unity. This is the position of the sphere when the source of sound is directly in front of the Rayleigh disk. The points indicated by small circles are the results of observations with the Rayleigh disk. The relative deflections are plotted. Our maximum deflection from the 45° position of the disk (*i. e.*, 45° between the normal to the mirror and the direction of the undisturbed stream) was 7°. An inspection of the formula[3] derived for the disk shows that the assumption of proportionality of energy to deflection does not introduce an error we need here consider.

[1] Zernov, Annal. d. Phys., No. 26, p. 79, 1908.

[2] Stewart, loc. cit.

[3] Konig, Wied. Annal., XLIII., 1891, p. 51.

TABLE I.

$kc = 1$, $kr = 2$.

	0°		30°		60°		90°	
	F	G	F	G	F	G	F	G
0	+0.2500	−.2500	+ .2500	−.2500	+.2500	−.2500	+.2500	−.2500
1	+ .7500	.0000	+ .6495	.0000	+.3750	.0000	.0000	.0000
2	+ .3020	+.2669	+ .1887	+.1669	−.0377	−.0334	−.1510	−.1335
3	+ .0949	+.1352	+ .0308	+.0439	−.0415	−.0591	.0000	.0000
4	+ .0398	+.0616	+ .0009	+.0014	−.0115	−.0178	+.0150	+.0231
5	+ .0188	+.0292	− .0042	−.0065	+.0017	+.0026	.0000	.0000
6	+ .0091	+.0142	− .0034	−.0053	+.0029	+.0046	−.0029	−.0044
	+1.4646	+.2571	+1.1123	−.0496	+.5389	−.3531	+.1111	−.3648
$F^2 + G^2$	2.2111		1.2397		0.4151		0.1454	

	120°		150°		180°	
	F	G	F	G	F	G
0	+.2500	−.2500	+.2500	−.2500	+.2500	−.2500
1	−.3750	.0000	−.6495	.0000	−.7500	.0000
2	−.0377	−.0334	+.1887	+.1669	+.3020	+.2669
3	+.0415	+.0591	−.0308	−.0439	−.0949	−.1352
4	−.0115	−.0178	+.0009	+.0014	+.0398	+.0616
5	−.0017	−.0026	+.0042	+.0065	−.0188	−.0292
6	+.0029	+.0046	−.0034	−.0053	+.0091	+.0412
	−.1315	−.2401	−.2399	−.1244	−.2629	−.0717
$F^2 + G^2$	0.0749		0.0730		0.0742	

The agreement between the theory of the acoustic shadow and the performance of the disk is not good in Fig. 3 for $kr = 2$, but is quite satisfactory in Fig. 4, $kr = 3$, conditions considered. If the disk itself were suspended in the open air it would give correct relative values of the mean kinetic energy per unit volume at the point. But the disk is enclosed in order to utilize the magnifying effect of resonance. This introduces several sources of error. A slight breeze interferes with the resonance of the tube, and doubtless this error has not the same relative value for all values of resonance. We worked under the best conditions obtainable and yet there was always a perceptible motion of the atmosphere. Our observations indicate that the small readings were greatly in error. It should be stated that the observed points are not averages of large numbers of readings, but represent different sets of observations. Another source of error is introduced by the absorption of energy by the resonating disk tube. This must disturb the distribution of sound intensity. It would seem that this distortion would tend to "iron out" the curve, or to produce higher readings on the steeper portions. One would also expect the distortion at $kr = 3$ to be less than at $kr = 2$.

TABLE II.

$kc = 1$, $kr = 3$.

	0°		30°		60°		90°	
	F	G	F	G	F	G	F	G
0	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500
1	+0.7000	+0.1000	+0.6062	+0.0866	+0.3500	+0.0500	0.0000	0.0000
2	+0.1311	+0.2904	+0.0819	+0.1815	−0.0164	−0.0363	−0.0656	−0.1452
3	−0.0122	+0.0876	−0.0039	+0.0285	+0.0053	−0.0383	0.0000	0.0000
4	−0.0082	+0.0212	−0.0002	+0.0005	+0.0024	−0.0061	−0.0031	+0.0079
5	−0.0026	+0.0057	+0.0006	−0.0013	−0.0002	+0.0005	0.0000	0.0000
6	−0.0008	+0.0017	+0.0003	−0.0007	−0.0003	+0.0006	+0.0003	−0.0005
	+1.0573	+0.2566	+0.9349	+0.0451	+0.5908	−0.2796	+0.1816	−0.3878
$F^2 + G^2$	1.1841		0.8760		0.4274		0.1834	

	120°		150°		180°	
	F	G	F	G	F	G
0	+0.2500	−0.2500	+0.2500	−0.2500	+0.2500	−0.2500
1	−0.3500	−0.0500	−0.6062	−0.0866	−0.7000	−0.1000
2	−0.0164	−0.0363	+0.0819	+0.1815	+0.1311	+0.2904
3	−0.0053	+0.0383	+0.0039	−0.0284	+0.0122	−0.0876
4	+0.0024	−0.0061	−0.0002	+0.0005	−0.0082	+0.0212
5	+0.0002	−0.0005	−0.0006	+0.0013	+0.0026	−0.0057
6	−0.0003	+0.0006	+0.0003	−0.0007	−0.0008	+0.0017
	−0.1184	−0.3040	−0.2709	−0.1824	−0.3131	−0.1301
$F^2 + G^2$	0.1065		0.1065		0.1150	

This expectation seems to be realized in the observations as shown in Figs. 3 and 4.

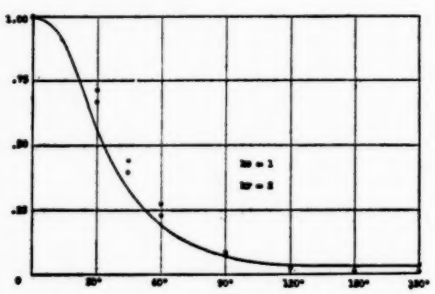

Fig. 3.

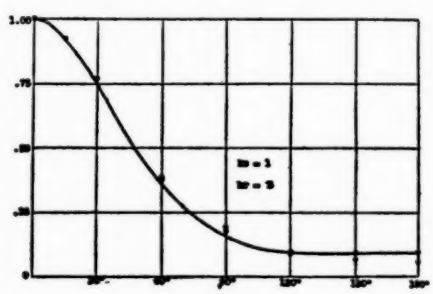

Fig. 4.

The theory, equation (2), shows that the intensities at different distances are proportional to $(F^2 + G^2)/r^2$. The results of a brief test at three different distances, $kr = 2$, $kr = 3$ and $kr = 4$, are presented in the accompanying table.

TABLE III.

kr	F^2+G^2	$A=\frac{F^2+G^2}{r^2}$.	B = Deflection.	Ratio of A to B.
2	2.206	0.551	11.80	21.4
3	1.145	0.127	2.80	22.0
4	0.915	0.0572	1.28	22.4

The theory is verified in that the ratio between the observed and theoretical relative values is practically constant. It is interesting to note that if the observed values are tested in a similar manner, but assuming the inverse square law, the results for the last column differ as much as 130 per cent.

The results presented in this paper certainly demonstrate that the method of producing known relative sound intensities is a practicable one, although attended with some difficulty of operation and limited both by the absorption of the instrument to be calibrated and the inconstancy of the source of sound. So far as the experiments with the Rayleigh disk are concerned, the results may be regarded in either of two ways. One may consider that they show the theory to be correct, assuming the deflection of the Rayleigh disk to be proportional to the energy. The writers, however, regard the theory of the acoustic shadow as more reliable than the assumption as to the action of the disk. The experiments call attention to certain errors in the operation of the disk which are unavoidable in any measuring instrument which utilizes resonance.

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THE POTENTIAL AND ELECTROSTATIC FORCE IN THE FIELD OF TWO METAL SPHERICAL ELECTRODES.[1]
III. CHARGE AND CAPACITY.

BY GEO. R. DEAN.

THE surface integral of normal force intensity over any closed surface drawn in the field is equal to 4π times the total charge of electricity inside the surface. When the charges are equal, all the lines of force start from one sphere and end on the other; they all cross the plane of zero potential at right angles to that plane. Hence, if G_0 denote the electrostatic force at any point of the plane, Q the charge on either sphere, and z the distance of the point from the common axis, we have

$$4\pi Q = \int_0^\infty 2\pi z \cdot G_0 \cdot dz. \tag{52}$$

The potential at any point (see 46, Part I.) is given by

$$V = \frac{V_1}{2} 2\sqrt{(\cosh \eta - \cos \xi)} \sum_0^\infty \frac{\sinh (m + \frac{1}{2})\eta}{\sinh (m + \frac{1}{2})\alpha} P_m(\cos \xi) e^{-(m+1/2)\alpha}, \tag{53}$$

the potentials of the spheres being $V_1/2$ and $-V_1/2$.
The electrostatic force at any point of plane of zero potential is

$$G_0 = \left(\frac{\partial V}{\partial r}\right)_{\eta=0} = \left(\frac{\partial V}{\partial \eta}\frac{\partial \eta}{\partial r} + \frac{\partial V}{\partial \xi}\frac{\partial \xi}{\partial r},\right)_{\eta=0}, \tag{54}$$

and since V is constant over the plane, this reduces to

$$G_0 = \left(\frac{\partial V}{\partial \eta} \cdot \frac{\partial \eta}{\partial r}\right)_{\eta=0},$$

r and z having same meaning as in § I.
Differentiating (1),

$$\frac{\partial V}{\partial \eta} = \frac{V_1}{2} \frac{\sinh \eta}{\sqrt{2(\cosh \eta - \cos \xi)}} \sum_0^\infty \frac{\sinh (m + \frac{1}{2})\eta}{\sinh (m + \frac{1}{2})\alpha} P_m(\cos \xi) e^{-(m+1/2)\alpha}$$

$$+ \frac{V_1}{2} \sqrt{2(\cosh \eta - \cos \xi)} \sum_0^\infty \frac{(m + \frac{1}{2})\cosh(m + \frac{1}{2})\alpha}{\sinh (m + \frac{1}{2})\alpha} P_m(\cos \xi) e^{-(m+1/2)\alpha} \tag{55}$$

[1] For Part I. and Part II. see THE PHYSICAL REVIEW, p. 459, December, 1912.

Then

$$\frac{\partial V}{\partial \eta}_{\eta=0} = \frac{V_1}{2}\sqrt{2(1-\cos\xi)}\sum_0^\infty \frac{(m+\frac{1}{2})P(\cos\xi)\ e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha}. \quad (56)$$

From (3), Part I.,

$$\eta = \tfrac{1}{2}\log\left[\frac{z^2+(c+r)^2}{z^2+(c-r)^2}\right];$$

$$\frac{\partial \eta}{\partial r} = \frac{c+r}{z^2+(c+r)^2} + \frac{c-r}{z^2+(c-r)^2},$$

and

$$\left(\frac{\partial \eta}{\partial r}\right)_{r=0} = \frac{2c}{z^2+c^2} = \frac{1-\cos\xi}{c}.$$

Then

$$G_0 = \frac{2V}{c}\sin^3\frac{\xi}{2}\sum_0^\infty \frac{(m+\frac{1}{2})P_m(\cos\xi)e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha} \quad (57)$$

For points in the plane of symmetry,

$$\tan\tfrac{1}{2}\xi = \frac{c}{z}.$$

Then

$$z^2 = c^2\cot^2\tfrac{1}{2}\xi,$$

and

$$2zdz = -c^2\cot\tfrac{1}{2}\xi\ \mathrm{cosec}^2\tfrac{1}{2}\xi\cdot d\xi.$$

When $z = 0$, $\xi = \pi$, and when $z = \infty$, $\xi = 0$. Substituting value of G_0 and $2zdz$ in (52),

$$4\pi Q = -2\pi c V_1 \int_0^\pi \cos\tfrac{1}{2}\xi dz \cdot \sum_0^\infty \frac{(m+\frac{1}{2})P_m(\cos\xi)}{\sinh(m+\frac{1}{2})\alpha} e^{-(m+1/2)\alpha},$$

$$= -2\pi c V_1 \sum_0^\infty \frac{(m+\frac{1}{2})e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha}\int_0^\pi P_m(\cos\xi)\cos\tfrac{1}{2}\xi d\xi. \quad (58)$$

We have now to evaluate the integral,

$$\int_0^\pi P_m(\cos\xi)\cos\tfrac{1}{2}\xi d\xi.$$

On page 9, Todhunter's Functions of Laplace, etc., we find

$$P_m(\cos\xi) = 1 - \frac{(n+1)n}{1^2}\sin^2\frac{\xi}{2} + \frac{(n+2)(n+1)n(n-1)}{1^2\cdot 2^2}\sin^4\frac{\xi}{2} - \cdots$$

by means of which we find

$$\int_0^\pi P_m(\cos\xi)\cos\tfrac{1}{2}\xi d\xi = \frac{2}{2m+1}.$$

Substituting in (58), we get

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$$G_0 = \frac{2V}{c}\sin^3\frac{\xi}{2}\sum_0^\infty \frac{(m+\frac{1}{2})P_m(\cos\xi)e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha} \quad (57)$$

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by means of which we find

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Substituting in (58), we get

$$4\pi Q = -2\pi c V_1\sum_0^\infty \frac{e^{-(m+1/2)\alpha}}{\sinh(m+\frac{1}{2})\alpha},$$

or

$$Q = cV_1 \sum_0^\infty \frac{1}{1 - e^{(2m+1)\alpha}}$$

$$= cV_1 \left\{ \frac{1}{1 - e^{\alpha}} + \frac{1}{1 - e^{5\alpha}} + \frac{1}{1 - e^{7\alpha}} + \cdots \right\}.$$

The value of α is given in Part I., x being distance between surfaces, and a the radius:

$$\alpha = \log_e \frac{\sqrt{x^2 + 4ax} + x}{\sqrt{x^2 + 4ax} - x},$$

and

$$2c = \sqrt{x^2 + 4ax}.$$

For the capacity between the spheres, whose difference of potential is V we have

$$C = \frac{Q}{V} = \frac{\sqrt{x^2 + 4ax}}{2} \left\{ \frac{1}{1 - e^{\alpha}} + \frac{1}{1 - e^{5\alpha}} + \frac{1}{1 - e^{7\alpha}} + \cdots \right\}.$$

This is also the capacity of a sphere of potential V against a conducting plane of potential *zero*, *i. e.*, capacity of sphere to ground. Here $x/2$ is the distance from plane to surface of sphere. If D be the distance from plane to center,

$$C = \sqrt{D^2 - 2aD} \left\{ \frac{1}{1 - e^{\alpha}} + \frac{1}{1 - e^{5\alpha}} + \frac{1}{1 - e^{7\alpha}} + \cdots \right\}.$$

The charge on the sphere $\eta = \alpha$ is given by

$$Q_\alpha = \frac{1}{4\pi} \int \left(\frac{\partial V}{\partial v} d\omega \right)_{\eta = \alpha}, \tag{59}$$

where dv is an elemental arc of the line of force, and $d\omega$ is the elemental area on the surface of the sphere. The element of the line of force coincides at the surface with the element of the circle ξ = constant. Hence, we have

$$\frac{\partial V}{\partial v} = \frac{\partial V}{\partial \eta} \cdot \frac{\partial \eta}{\partial v}, \tag{60}$$

$\partial V/\partial \xi$ being zero at that point.

From (20), Part I.,

$$x = \frac{c \sinh \eta}{\cosh \eta - \cos \xi}, \quad y = \frac{c \sin \xi}{\cosh \eta - \cos \xi}. \tag{61}$$

Taking ξ constant,

$$dx = \frac{c(1 - \cosh \eta \cos \xi)}{(\cosh \eta - \cos \xi)^2}, \quad dy = \frac{c \sinh \eta \sin \xi}{(\cosh \eta - \cos \xi)^2}. \tag{62}$$

Then

$$dv = \sqrt{(dx)^2 + (dy)^2} = \frac{cd\eta}{\cosh \eta - \cos \xi},$$

and

$$\frac{\partial \eta}{\partial v} = \frac{\cosh \eta - \cos \xi}{c}. \tag{63}$$

For the element area of the surface, we have

$$d\omega = 2\pi y d\sigma, \tag{64}$$

$d\sigma$ being an element of a great circle.

Taking η constant, we find, by means of (61)

$$d\sigma = \frac{cd\xi}{\cosh \eta - \cos \xi}, \tag{65}$$

$$d\omega = \frac{2\pi c^2 \sin \xi d\xi}{(\cosh \eta - \cos \xi)^2}, \tag{66}$$

$$\frac{\partial \eta}{\partial v} \cdot d\omega = \frac{4\pi c \sin \xi d\xi}{2(\cosh \eta - \cos \xi)}, \tag{67}$$

$$\left(\frac{\partial \eta}{\partial v} d\omega\right)_{\eta=\alpha} = \frac{4\pi c \sin \xi d\xi}{2(\cosh \alpha - \cos \xi)}. \tag{68}$$

When the potential of the sphere $\eta = \alpha$ is V_1, and that of the sphere $\eta = -\alpha$ is zero, the potential at any point is given by

$$V = V_1 \left\{ \sqrt{2(\cosh \eta - \cos \xi)} \sum_0^\infty \frac{\sinh (m+\frac{1}{2})(\alpha+\eta)}{\sinh (m+\frac{1}{2})(2\alpha)} P_m(\cos \xi) e^{-(m+1/2)\alpha}. \right. \tag{69}$$

$$\frac{\partial V}{\partial \eta} = V_1 \left[\frac{\sinh \eta}{\sqrt{2(\cosh \eta - \cos \xi)}} \sum_0^\infty \frac{\sinh (m+\frac{1}{2})(\alpha+\eta)}{\sinh (2m+1)\alpha} P_m(\cos \xi) e^{-(m+1/2)\alpha} \right.$$

$$+ \sqrt{2(\cosh \eta - \cos \xi)} \sum_0^\infty \frac{\cosh (m+\frac{1}{2})(\alpha+\eta)}{\sinh (2m+1)\alpha} \tag{70}$$

$$\left. \times (m+\tfrac{1}{2}) P_m(\cos \xi) e^{-(m+1/2)\alpha} \right],$$

$$\left(\frac{\partial V}{\partial \eta}\right)_{\eta=\alpha} = V_1 \left[\frac{\sinh \alpha}{\sqrt{2(\cosh - \cos \xi)}} \sum_0^\infty P_m(\cos \xi) e^{-(m+1/2)\alpha} \right.$$

$$+ \sqrt{2(\cosh \alpha - \cos \xi)} \tag{71}$$

$$\left. \sum_0^\infty (m+\tfrac{1}{2}) \coth(2m+1)\alpha \cdot P_m(\cos \xi) e^{-(m+1/2)\alpha} \right],$$

$$\left(\frac{\partial V}{\partial \eta}\right)_{\eta=-\alpha} = V_1 \left[\sqrt{2(\cosh \alpha - \cos \xi)} \right.$$

$$\left. \times \sum_0^\infty (m+\tfrac{1}{2}) \coth(2m+1)\alpha P_m(\cos \xi) e^{-(m+1/2)\alpha} \right], \tag{72}$$

Now

$$\sum_0^\infty P_m(\cos\xi)e^{-(m+1/2)\alpha} = \left\{2(\cosh\alpha - \cos\xi)\right\}^{-1/2}.$$

Therefore

$$\left(\frac{\partial V}{\partial \eta}\right)_{\eta=\alpha} = V_1\Big[\frac{\sinh\alpha}{2(\cosh\alpha-\cos\xi)} + \sum_0^\infty (m+\tfrac{1}{2})\coth(2m+1)\alpha \times e^{-(m+1/2)\alpha}P_m(\cos\xi)\sqrt{2(\cosh\alpha-\cos\xi)}\Big]. \tag{73}$$

To find Q_α, we have then to evaluate the following integrals:

$$\int_0^\pi \frac{\sin\xi d\xi}{[2(\cosh\alpha-\cos\xi)]^2} = -\tfrac{1}{4}\Big[(\cosh\alpha-\cos\xi)^{-1}\Big]_0^\pi = \frac{1}{2\sin^2 h\alpha}; \tag{74}$$

$$\int_0^\pi \frac{P_m(\cos\xi)\sin\xi d\xi}{\sqrt{2(\cosh\alpha-\cos\xi)}} = \int_0^\pi \sum_0^\infty e^{-(\eta+1/2)\alpha}P_\eta(\cos\xi)P_m(\cos\xi)\sin\xi d\xi. \tag{75}$$

$$\int_0^\pi [P_m(\cos\xi)]^2(\sin\xi d\xi) = \int_{+1}^{-1} -[P_m(x)]^2dx = +\frac{2}{2m+1};$$

$$\int_0^\pi P_n(\cos\xi)P_m(\cos\xi)\sin\xi d\xi = -\int_{-1}^{+1} P_n(x)P_m(x)dx = 0.$$

Therefore (75) becomes

$$\int_0^\pi \frac{P_m(\cos\xi)\sin\xi d\xi}{\sqrt{2(\cosh\alpha-\cos\xi)}} = \frac{2e^{-(m+1/2)\alpha}}{2m+1}.$$

Accordingly, substituting in (59), we find

$$Q_\alpha = cV_1\Big[\operatorname{cosech}\alpha + 2\sum_0^\infty \coth(2m+1)\alpha\cdot e^{-(m+1/2)\alpha}\Big],$$

$$Q_{-\alpha} = 2cV_1\sum_0^\infty \operatorname{cosech}(2m+1)\alpha\cdot e^{-(m+1/2)\alpha}.$$

OPTIMUM WAVE-LENGTH IN RADIOTELEGRAPHY.

By A. H. Taylor.

GIVEN a definite maximum of available power, and a fixed aerial system, the question as to a suitable choice of wave-length presents itself. The fairly well established facts influencing the choice of wave-length are as follows: First, the radiation of the aerial is greater for short waves. Second, the aerial current with a given potential maximum is greater for short waves. Third, the absorption is greater for short waves than for long ones. Fourth, the absorption increases with the distance.

Austin has investigated the formula[1] $I = \frac{kh_1h_2I_1e^{-\alpha D/\sqrt{\lambda}}}{\lambda D}$, where

h_1 = height of sender, h_2 = height of receiver,
λ = wave-length, I_1 = sending aerial current,
I_2 = received current through 25 ohms equivalent resistance,
D = distance between stations,
α = .0015 for transmission over salt water.

The lengths are all in kilometers and the currents in amperes. The constant K has been determined by Austin to be 4.25 for the flat-topped aerials on the cruisers used in his experiments.

As far as I am aware, no publication has been made of similar long distance experiments over land, from which K and α could be determined for any given character of soil and type of aerial. It is known that the absorption over land[2] is in general greater, although the surface absorption of the earth[3] has not been separated from the atmospheric absorption which produces the great difference between day and night messages.

The constant K would depend on local conditions at sending and receiving stations, and for properly located land stations should probably be a little larger than 4.25. The other factors of the formula have been found to be nearly correct for overland transmission and it is natural to assume that the received current will follow the same general law of variation over land distances as over the sea, especially over the flat prairie where the author's experiments are being undertaken.

[1] Austin, Bull. Bur. St., Vol. 7, No. 3, 1911.
[2] Zenneck, Hun. d. Phys., 23, 1907.
[3] Pierce, Wireless Telegraphy, p. 131, 1910 edition.

It has been shown by Austin[1] and others that within practical ranges of usefulness, nearly all detectors respond approximately in such a manner that the audibility A of the signal is proportional to the square of the received current. This makes the audibility inversely proportional to D^2, and directly proportional to i_1^2/λ^2, neglecting the absorption. The installation at the University of North Dakota is at present limited to 1 kw. output, furnished by a Type E self-controlling transformer on a 110-volt 60-cycle circuit. The primary condenser usually had a capacity of .02 Mf. and was coupled inductively to the aerial, loosely enough so

TABLE I.

λ = .400 km., C_1 = .01 Mf.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.220	2.1	138
.320	2.6	132
.400	2.85	128
.500	3.1	120
	Average..130	

TABLE II.

λ = .457 km., C_1 = .02 Mf.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.370	3.3	141
.450	4.0	169
	4.3	143
87[illegible]	5.7	177
	Average..157	

TABLE III.

λ = .560 km., C_1 = .03 Mf.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.580	3.8	79
.685	4.2	82
.850	4.8	86
	Average...82	

TABLE IV.

λ = .590 km., C_1 = .03 Mf.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.570	3.75	70.5
.700	4.2	72
.800	4.6	76
1.050	5.4	76
	Average..74	

TABLE V.

λ = .660 km., C_1 = .02 Mf.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.300	3.0	69
.380	3.4	69.5
.460	3.65	66.5
.620	3.9	56
	Average..65	

TABLE VI.

λ = .730 km., C_1 = .01 Mf.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.252	2.35	41
.400	3.25	49.5
.500	3.45	44.6
.600	3.7	43
	Average..44.5	

TABLE VII.

λ = .390 km. Free aerial operated by induction coil on D.C.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.140	2.85	382
.120	2.6	372
.065	1.7	315
	Average..356	

TABLE VIII.

λ = .390 km. Induction coil on A.C. 60-cycle.

P Kilowatts.	I Aerial Current.	$i^2/\lambda^2 P$.
.037	1.35	323

TABLE IX.

Wavelength.	Av. $i^2/\lambda^2 P$.	i/λ for P = 1Kw.
.730	44.5	6.65
.660	65.	8.06
.590	74.	8.6
.560	82.	9.05
.457	157.	12.55
.400	130.	11.4

[1] Austin, Bull. Bur. St., Vol. 6, No. 4, 1910.

that the two waves transmitted did not differ more than 7 per cent. The aerial was of the inverted L type, 27 meters high, 3 meters wide, 40 meters long, of 6 No. 14 aluminum wires. The vertical lead was a single No. 12 stranded copper wire. Its free wave-length was 390 meters. The following tables show the dependence of the factor i_1^2/λ^2 upon the power and upon the wave-length.

There was some irregularity in the action of the zinc spark gap which may account for the variation in the factor $i^2/\lambda^2 \cdot P$, but as a first approximation I propose to call this factor a function of λ but not of P. It represents the intensity or energy radiation factor for a given wave-length and determines the audibility of the signals received at short distances, where the absorption may be neglected. For minimum energy required to produce audible signals at moderate distances, it is evident that the wave-length .457 km. is best. This conclusion has been verified by an operator at Inkster, N. D., thirty-five miles distant, receiving messages sent from the university at 100 watts power.

It is hardly fair to include the data for the free aerial, operated by the secondary of an induction coil directly connected to the aerial, since the maximum amount of energy which can be radiated by the system is reached in the case of direct current and interrupter at 140 watts and with alternating current at 65 watts. This low limit is due in the latter case to arcing at the spark gap and in the former case to limiting insulation. The very high value of $i^2/\lambda^2 \cdot P$ which is obtained by this simple coupling recommends it for very short range work, not requiring much power. It is interesting to note that this factor is not very different for D.A. and A.C. although the spark frequency is much higher for A.C.

In regard to the data on the coupled aerial it is to be noted that at $\lambda = .400$ km. there were so few turns used on the secondary helix that the degree of coupling was insufficient. The radiant efficiency factor increases to a maximum and then falls off rapidly as the wave-length approaches the free wave-length of the aerial where the coupling is zero.

Austin puts $I_2 = 4 \times 10^{-5}$ through a receiving aerial of 25 ohms equivalent resistance as sufficient for good communication. Assuming this value we have

$$D = \frac{4.25 k_1 h_2 \epsilon \, e^{\frac{-4D}{\sqrt{\lambda}}} i_1}{4 \times 10^{-5} \quad \lambda}$$

for the maximum distance for good working. This can be readily solved by successive approximations, and it is at once evident that this distance is not necessarily greater for the short waves which give the strongest signals at closer ranges. The factor i_1/λ, determined for the maximum

power of the station, may be regarded as the *distance factor*, since it would determine the maximum distance through non-absorbing space, as approximately, for night transmission at sea. For such transmission the shortest wave possible with proper coupling will again give the best result. Also, if the heights h_1 and h_2 are not sufficient to give a range of over say 200 km. the absorption factor $\epsilon^{-\alpha D/\sqrt{\lambda}}$ will be so small that the distance factor i/λ, greatest for short waves, will control the result. The same argument holds for very low power stations. As a sample calculation, the working distance of this station, sending at 1 kw. to a similar aerial is calculated for different observed values of i/λ at different wave-lengths.

Assuming temporarily $K = 4.25$, $\alpha = .0015$:

λ	i/λ maximum	D (K. M.)
.457	12.53	343
.560	9.05	320
.590	8.66	312
.660	8.06	307
.730	6.65	272

It will be noted that the sending range for 1 kw. is the maximum obtainable with the shortest practicable wave-length, 460 meters. $\lambda = .450$ to .600 probably would give very nearly the same working range for this station to a similar one. It is to be noted that the wave-length $\lambda = .457$, for which $i^2/\lambda^2 \cdot P = 157$ will give at close range twice the audibility of $\lambda = .590$, for which $i^2/\lambda^2 \cdot P = 74$, although their maximum working distance is not much different.

If we assume with Austin that 1×10^{-5} amperes give the limit of extreme audibility, the wave-length for maximum distance is still .457, as Table XI. shows.

λ	D
.457	702
.560	640
.590	648
.660	643
.730	617

Table XII. gives the variation of extreme range of audibility with wave-length for $P = 4$ kw., on the assumption that the quantity $i_1^2/\lambda^2 \cdot P$ is constant for λ constant, and that i_1/λ is proportional to $\sqrt{P}$.

λ	D
.457	905
.560	860
.590	859
.660	865
.730	828

These conditions could probably be realized closely with a transformer of the same type, but of higher power. There is, within the limits of experimental error, but little preference for any wave between $\lambda = .457$ and $\lambda = .660$, although the audibilities at close range have the ratio $157/65 = 2.4$ in favor of $\lambda = .457$.

On increasing the height of the aerials longer waves will obviously be necessary, but the concomitant increase in power and range will also throw the optimum towards longer wave-lengths. As far as this station is concerned, it is evident that the shortest wave-length consistent with good coupling gives the best result under all conditions, *unless* the absorption is very much greater than the assumed value, in which case a somewhat longer wave will be better. Experiments are now under way which will test these points as far as is possible for a station of this low power.

University of North Dakota.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-SIXTH MEETING.

A REGULAR meeting of the Physical Society was held in the Sloane Physical Laboratory of Yale University, New Haven, on Saturday, March 1, 1913, with President Peirce in the chair.

The following papers were presented:

The Actinodielectric Effect of Sulphur. CHESTER A. BUTMAN.

Note on the Ageing Effect as an Influence on the Number and the Velocities of Photo Electrons from Thin Cathode Films. OTTO STUHLMANN, JR.

A Powerful Source of Ultraviolet Light for Photo-electric Work. ALBERT W. HULL AND ANCEL ST. JOHN.

On the Beaded Character of the Deflected Cathode Ray Line as Revealed by Instantaneous Photographs taken at Short Range. CHARLES T. KNIPP.

Dynamic Characteristics of the Wehnelt Rectifier. R. A. PORTER.

The Dimensions of Electric and Magnetic Quantities. DAVID L. WEBSTER.

The Forced Vibration of a Circular Plate. A. G. WEBSTER.

On the Occurrence of Hydrogen Lines in the Spectrum of the "Tube-Arc," and Application to the Nature of Enhanced Lines. ARTHUR S. KING.

The Dispersion of Metals. LYNDE P. WHEELER.

On a Fundamental Law of Mechanics. H. M. DADOURIAN.

A New Vacuum Gauge of Extreme Sensitiveness. I. LANGMUIR.

The Extinguishment of Phosphorescent Light. CHESTER A. BUTMAN.

Adjourned at 1 P. M.

ERNEST MERRITT,
Acting Secretary.

ON THE BEADED CHARACTER OF THE DEFLECTED CATHODE RAY LINE AS REVEALED BY INSTANTANEOUS PHOTOGRAPHS TAKEN AT SHORT RANGE.[1]

BY CHAS. T. KNIPP.

J. J. THOMSON[2] in a recent article on "Positive Rays" calls attention to two kinds of beading. One he styles atomic, and the other is found on the molecular lines. The first is explained by some of the atoms having a

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Phil. Mag., Vol. 24, p. 235, Aug., 1912.

double charge. This is shown on the photographic plate as a beak or hook-like projection extending beyond the head of the parabola and towards the vertical axis. The second, the molecular beading, extends away from the vertical axis. His photographs often show as many as three and four beads on a single molecular line. As to the explanation of the latter he is not so sure, but suggests two possible causes. The most likely of the two is that the cathode rays by impact on the walls of the discharge vessel give rise to secondary rays of great ionizing power, producing positive carriers having less energy than those which started from the negative glow.

Recently while photographing retrograde rays the writer was impressed with the beaded character of the cathode ray line in many of the photographs. The plate was mounted in the apparatus so that it could be revolved. Thus instantaneous photographs were obtained, each impression on the plate being due nominally to a single discharge of the induction coil. The cathode rays after passing through the tube (3 cm. long by about .2 mm. internal diameter), were deflected by a strong electrostatic field. The distance of the photographic plate from the mouth of the tube was 1.62 centimeters.

Under these conditions the instantaneous photographs generally reveal a central undeflected spot, and *one*, *two*, and in several instances, *three* and *four* spots all in a straight line along the direction of the electrostatic field, which in this instance was horizontal. Time exposures show the central spot clearly, and a continuous though in general beaded line. The positions of the spots on the instantaneous photographs are more or less rhythmical and if superposed, as in a time exposure, may readily produce the beaded line.

The explanation given by Thomson for the atomic carriers does not apply in this case, for we are dealing with the elementary charge. While the secondary ray explanation may apply, it seems to the writer that the true explanation for the case of the beaded cathode ray line is to be found in the oscillatory character of the electric discharge. Clearly the rays due to subsequent oscillations would fall off in energy and be recorded on the photographic plate as groups of electrons moving with successively diminished velocities.

A number of photographs accompany the paper.

DYNAMIC CHARACTERISTICS OF THE WEHNELT RECTIFIER.[1]

BY R. A. PORTER.

THE Wehnelt rectifier is a highly evacuated discharge tube, one electrode of which is glowing calcium oxide, while the other may be of any kind of conductor. In a series of articles Wehnelt[2] has described and explained

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] A. Wehnelt, Verh. d. D. Physik. Ges., 5, pp. 255–258, 1903; Ann. d. Phys. (4), 14, pp. 425–468, 1904; (4) 19, pp. 138–156, 1906; Physik. Zeitschr., 9, pp. 134–135, 1908; A. Wehnelt and F. Jentzsch, Verh. d. D. Physik. Ges., 10, pp. 605–615, 1908; Ann. d. Phys. (4), 28, pp. 537–552, 1909.

many of the peculiarities of such a discharge tube. The voltage-current curves for voltages that have been applied long enough for equilibrium to become established, *i. e.*, the "static characteristics" of this tube, have been described by S. Marsh.[1] Although this tube is of special interest as a rectifier of alternating currents, there has been heretofore no investigation of the current-voltage curves for such a current, *i. e.*, of the "dynamic characteristics." Simon[2] has shown that in the case of other conductors these two sets of curves are not the same.

The dynamic characteristics were obtained from oscillograph records by measuring corresponding instantaneous values of current through the discharge tube and voltage between the terminals of the tube. From such records dynamic characteristics were plotted which show the effects produced respectively by variation in the applied effective voltage, the series resistance, the pressure in the tube, the temperature of the oxide electrode, the atmosphere in which the discharge occurs, and the frequency of the applied voltage.

The results may be summarized as follows: (1) The curve which shows the relation between applied voltage and current for the half-period during which luminous discharge takes place forms a loop (hysteresis). The luminous discharge begins at a certain voltage, called the ignition voltage; the characteristic then follows closely the path of the static characteristic of the glow discharge, returns from the point of maximum current at first along a line close to the curve for increasing current, but at smaller current values the voltage is considerably lower. (2) Within the range studied, the ignition voltage decreases as the pressure increases. (3) The ignition voltage increases with increase of the frequency of the applied A. C. voltage. (4) At low pressures, 0.001–0.003 mm., the characteristics are similar, whether the tube filling be of air, hydrogen, or oxygen. (5) At this low pressure the presence of mercury vapor in the discharge tube lowers all voltages to about one third the values which they have when mercury is excluded. (6) At larger pressures, 0.03–0.05 mm. the characteristics for all four gases are approximately the same. (7) When a constant voltage sufficiently high to produce luminous discharge has been applied to the tube, a definite length of time must elapse before luminous discharge begins. The length of this delay depends on the voltage and pressure; it is independent of the temperature of the cathode. (8) Observations with iodine as an atmosphere show an anomalous effect, in that the hysteresis under certain conditions is negative, and when a constant direct-current voltage is applied a perfectly regularly interrupted current may be obtained.

The peak in the voltage curve and the hysteresis effect can be explained on the ground that a certain length of time must elapse before a new potential gradient in the tube can be established.

[1] S. Marsh, Dissertation, Göttingen, 1909.

[2] H. Th. Simon, Physik. Zeitschr., 6, pp. 297–319, 1905; 7, pp. 433–445, 1906.

The phenomena of the delay in ignition and of the production of an interrupted current by iodine are being further investigated.

SYRACUSE UNIVERSITY,
SYRACUSE, N. Y.,
February 4, 1913.

THE DIMENSIONS OF ELECTRIC AND MAGNETIC QUANTITIES.[1]

BY DAVID L. WEBSTER.

AS it is well known, any attempt to assign to electric or magnetic quantities dimensions expressed in terms of mass, length, and time, leads to fractional exponents in the dimensional formulas. For example, electric charge has the dimensions $m^{\frac{1}{2}}l^{\frac{3}{2}}t^{-1}$ in the electrostatic system, or $m^{\frac{1}{2}}l^{\frac{1}{2}}$ in the electromagnetic system, the difference being "explained" by the unknown dimensions of the electric and magnetic susceptibilities.

To remove the difficulty of interpretation of these fractional exponents, it is proposed to abandon mass as a fundamental concept, replacing it by quantity of electricity (e), and taking electric susceptibility, as indicated by the electron theory, to be a pure number. From this assumption, with the inverse square law of electrostatics, one may readily obtain the following dimensions: electric force, el^{-2}; potential, el^{-1}; current et^{-1}; resistance, $l^{-1}t = v^{-1}$; etc. And with the aid of Biot and Savart's law one obtains: magnetic force, el^{-2}; pole strength e; etc. And introducing Newton's laws of motion, one obtains: mechanical force, e^2l^{-2}; mass, $e^2l^{-3}t^2 = e^2l^{-1}v^{-2}$; etc.

This last formula suggests at once the concept of electromagnetic mass, the idea that all mass is of that nature, and the proportionality of the mass of an electron, directly to the square of its charge, and inversely to its radius and the square of the velocity of light.

A further simplification is introduced if time is considered as merely the distance that light may travel, in which case each t in the formulas is replaced by l, and the v's are all omitted.

A POWERFUL SOURCE OF ULTRAVIOLET LIGHT FOR PHOTO-ELECTRIC WORK.[1]

BY ALBERT W. HULL AND ANCEL ST. JOHN.

THE source of light used is a vacuum discharge tube of the general form used by Lyman in his vacuum spectrograph, containing hydrogen at a low pressure. Experiments were made to find how the photo-electric efficiency depended on (1) length of capillary; (2) diameter of capillary; (3) pressure of gas; (4) current density in tube; (5) potential and frequency applied to tube.

The results may be summarized as follows:

(1) The efficiency increases slowly with length of capillary (50 per cent. increase in efficiency for 300 per cent. increase in length) in accordance with

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

the known absorption of ionized hydrogen. (2) The diameter of capillary, between the limits of 3 and 13 mm. makes no difference if the current is kept constant. (3) Pressure of the gas, between 2 and 15 mm., makes no difference if current is constant. (4) A strictly linear relation between current (in discharge tube) and photo-electric effect was found. (5) Various forms of interrupters and a powerful Tesla coil gave comparatively small photo-electric effects, corresponding to the smallness of current which they furnished. A transformer on a 110-volt A. C. circuit gave currents of 0.2 amp. through the tube, and produced a photo-electric current of 1.3×10^{-6} amp. per cm.^{2} on a platinum electrode in a vacuum of 1/1000 mm. For comparison, a mercury-in-quartz lamp was set up at a distance of 10 cm. from the plate, and when running steadily at 3.2 amp. gave a photo-current of 5.5×10^{-9} amp./cm.^{2} The hydrogen tube, therefore (with fluorite window), gives a photo-electric effect 250 times as large as the mercury arc.

WORCESTER POLYTECHNIC INSTITUTE,
February 14, 1913.

NOTE ON THE AGING EFFECT AS AN INFLUENCE ON THE NUMBER AND THE VELOCITIES OF PHOTO-ELECTRONS FROM THIN CATHODE FILMS.[1]

BY OTTO STUHLMANN, JR.

WHILE investigating the photo-electric properties of thin films deposited from a platinum cathode, for difference in photo-electric effect caused by emergent (E) and incident (I) light,[2] the writer found that films whose thickness were comparable with 10^{-7} cm., showed a gradual increase in the ratio of the emergent to the incident effect (E/I) as time went on. In one case this ratio E/I for a particular platinum film successively changed from 1.08 to 1.10 to 1.12 in three succeeding days. It gave very erratic and unreliable results, averaging 1.07 on the day previous to the taking of the first of the above values. Under normal conditions, the ratio E/I increases to a constant value as the thickness of the film decreases. It therefore seems probable that the film undergoes some sort of change as time goes on. Just after sputtering it was seemingly in an unstable state, gradually decreasing in thickness. Three days afterward it reached a stable state, giving a continuous value for $E/I = 1.12$.

Dike[3] in a recent paper discussing the velocities of photo-electrons emitted from films of platinum, similarly deposited as above, found that the velocities increased as the thickness of the film decreased. He also found that upon testing a film of platinum for 24 consecutive hours or more after depositing, he attained with constant illumination, the unusual result of a continuous increase in the velocities of the emitted electrons, as time went on. Since

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Phil. Mag., Aug., 1910, p. 331; Pt. II., Dec., 1911, p. 854.

[3] PHYS. REVIEW, XXXIV., p. 459, 1912.

an increase in velocity of the emitted electrons is accompanied by a decrease in thickness of the emitting film, it seems plausible that the aging of the film was accompanied by a decrease in its thickness. This would account for the resulting increase in velocity as time went on.

Let us examine why the above explanations of decrease in thickness of cathode films with time is the most probable one. The films are obtained by particles of the metal striking the glass or quartz surface and adhering. These particles are more or less loosely connected and can be considered as more or less irregular spheres packed into a semitransparent layer. If the discharge is very rapid the film becomes very spongy and can easily be removed by wiping with a finger. A slow discharge, on the other hand, produces a more compact and dense surface. Patterson[1] found, while working on the specific resistance of similar metal films, that heating them for some time by an electric current reduced their resistance in some cases more than 50 per cent., although he found it impossible to reduce the resistance to that of ordinary platinum. He also showed that heating them in a current of dry air or allowing them to lie idle for some days produced the same effect but not in so marked a degree.

Evidently, by heating, the particles settle into a more and more compact mass and consequently their resistance decreases.

Beilly[2] found that heating metal films at a temperature much below the melting point, conferred sufficient freedom on the molecules to enable them to behave as the molecules of the liquid metal would do. They arrange themselves under the influence of surface tension either in films or under certain other conditions into drop-like granular forms.

Evidently, allowing the films to age gradually has the same effect but not to so marked a degree as aging them rapidly by passing a heating current through the films. Through aging the semi-transparent granular structure will gradually sink into a more and more homogeneous layer and gradually decrease in thickness as the aging continues.

Evidently aging decreases the thickness of the films.

The ratio of the emergent to the incident photo-electric effect increases with the aging of the film, hence the decrease in thickness of the film would account for the results quoted.

The velocity of the emitted electrons increases with aging. Here again, aging decreasing the thickness of the film accounts for the gradual increase in velocities of the electrons, as time goes on.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.

[1] Phil. Mag., Dec., 1902, p. 666.
[2] Pro. Roy. Soc., 72, 1903, p. 226.

TRIAL OF A METHOD FOR OBTAINING EQUALITY OF TEMPERATURE, AT HIGH TEMPERATURES.[1]

BY C. F. LORENZ.

IN the simplest case two filaments, say wires of tungsten and platinum drawn through the same die, are similarly mounted in a gas-chamber which communicates with a sensitive aneroid pressure-indicator. Resistances in circuit with the two filaments are adjusted until there is no alteration of pressure when the filaments are switched on alternately, and equality of pressure is taken to indicate equality of temperature. Though the loss of energy of a filament through radiation is proportional to the diameter of the filament, the loss of energy through the cooling action of the surrounding gas is not proportional to the diameter, but varies much more slowly than the latter; this fact makes relatively large percentage errors in equality of diameters permissible, which is especially important when one or the other of the substances compared is one that is not capable of being drawn through a die, as, for example, carbon.

A comparison of tungsten and platinum was made with the result that these substances are very closely at a color match when they are at indicated equality of temperature.

PHYSICAL LABORATORY,
NATIONAL ELECTRIC LAMP ASSOCIATION,
CLEVELAND, O.

THE SERIES SYSTEM IN THE SPECTRA OF CA, SR, AND BA.[1]

BY F. A. SAUNDERS.

IN each element there are systems of single-line series, of pair series, and of triplet series. In the two latter systems both wide and narrow groups occur.

In regard to the single-line series, recent photographs in Ca have shown that the series formerly given as $SL1$[2] can be obtained with heavy reversals throughout, like a true principal series. In fact, in one case, the four last lines on the photograph were shown *as reversals only*, the emitted light of the separate series lines having fused into a continuous spectrum. More accurate measurements have been obtained of these lines, and strong arguments can be advanced for including in this series the lines 4,226.91 and 2,721.76, the first of which is the well-known flame line. Similar remarks apply to Sr. Using the Ba flame line 5,535.69 as the first line of Ba $SL1$, the others appear to be at 3,071.71, 2,702.78, 2,597.26, 2.543.3. Three of these give another series (Ba $SL2$) by a wave-number shift of 16,781.7, whose wave-lengths are 6,341.88, 4,947.50, and 4,605.11, though further work is required on this series. The formulæ for all these series are complicated; the denominator contains a function of

[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

[2] Astrophysical Journal, 32, 153, 1910.

the variable integer which is certainly more complicated than in the Ritz formula.

The system of pairs has been enlarged by the discovery (by Lyman) of ultraviolet lines, which form narrow pair series. These have the line of greater wave-length the stronger, as do typical "subordinate" series; and the lines are easily reversed. The pairs have constant wave-number differences, whose values are 65 for Ca, 280 for Sr, and 575 for Ba. A new relation apparently holds between these narrow pairs (p), and the wide pairs (P). The convergence wave-number of the complex pair series (P_1) *plus* the wave-number of the first term, equals the convergence wave-number of the narrow pair (p) series. This relation is true within the limits of accuracy of location of the convergence wave-numbers, though this accuracy is not yet very great. The form of the relation is similar to that for the narrow pair series in the alkalies, announced by Runge, except that the "plus" in the above statement becomes minus in that case. Furthermore, the wave-number-differences in the p series are equal to those occurring in the first term of the P_1 series (which consists of three lines), between the two lines of *shorter* wave-length. This law is fulfilled to a satisfactory degree of accuracy, and is again similar to a relation found by Runge, excepting that in the alkalies (*e. g.*, in Cs) the separation of the narrow pairs is equal to that of the two lines of *longer* wave-length in the first term of wide pairs.

Grateful acknowledgment is due to Dr. H. M. Randall for permission to use some unpublished results on Ba. The last relation above was used to predict a Ba line, later found by him at λ 12,085, which is probably the line of greatest wave-length in the first term of Ba P_1.

If we call an ordinary "principal series of pairs" a complex series of single lines (as we reasonably may), the Rydberg-Schuster law and the two laws above mentioned may be grouped in one general form, somewhat as follows: In all complex series, the convergence wave-number minus (or plus) the wave-number of the first term equals the convergence wave-number of a derived series, and the wave-number-difference of the lines of this derived series is constant and equal to that of the lines of the first term of the complex series (or certain ones of them). This applies then (*a*) to the ordinary sharp (second subordinate) series of pairs in the alkalies, etc., which are derived from the principal series (Rydberg-Schuster law); (*b*) to the narrow pair series in the alkalies, derived from the complex pair series (law of Runge, "Bergmann" series, etc.), and to the narrow triplet series in Ca and Sr (Ba still doubtful) derived from the complex triplet series; and, (*c*) finally, to the ultraviolet p series in Ca, Sr and Ba, derived from the complex pair series.

THE CONSTANTS OF SPECTRAL RADIATION OF A UNIFORMLY HEATED ENCLOSURE.[1]

BY W. W. COBLENTZ.

IN a previous communication to this society[2] a general description was given of the work which was completed prior to December, 1912. It was then proposed to observe several more sets of energy curves, at the highest operable temperatures with a vacuum furnace before publishing the exact numerical results. Owing to unavoidable delays in assembling all the apparatus, the intention of withholding all the data until the completion of this investigation has now been abandoned, and in the present paper is given a brief summary of the most reliable data now at hand.

The temperature scale used in this investigation, for temperature above 1400° C. is the optical scale, which is obtained by adding to the thermoelectric temperatures[3] certain values, which are the mean of several groups of thermocouples. In assembling the present data for publication it was observed that for temperatures above 1400° C. there was an abrupt rise in the value of the constant C which increased systematically to abnormally high values for temperatures taken above 1500°. It was therefore necessary to choose between the possibility that (1) this variation in C at high temperature is owing to the failure of Planck's law at high temperatures or (2) that, for the particular thermocouples used in this research the aforementioned correct factors for reduction to the optical scale are too large.[4] The writer chose the latter conclusion and the results obtained at high and at low temperatures are now in agreement.

The systematic errors, which formerly caused the mean values of the various sets of observations to differ by 0.2 to 0.5 per cent., are now reduced to less than 0.2 per cent. They were caused by the presence of a disproportionate number of high temperature energy curves, in some sets of data, which contained the aforementioned corrections for reduction to the optical temperature scale. This modification of the correction for reduction of the thermoelectric temperatures to the optical scale has no effect upon the data obtained in 1912, owing to the fact that, intentionally (to save the thermocouple calibration; the difficulty with the scale correction was not foreseen), no energy curves were observed at these high temperatures. The former unexplained disagreement between the sets of observations made in 1911 and those of 1912 is now reduced to an excellent agreement. This was to be expected, for there was no apparent difference in the methods of operation other than the addition

[1] Abstract of a paper to be presented at the Washington meeting of the Physical Society, April 26, 1913.

[2] Coblentz, PHYS. REV., 1, p. 249, 1913.

[3] Waidner and Burgess, Bull. Bur. Standards, 3, p. 205, 1907.

[4] In the complete paper it will be shown that, for the particular thermocouples, the mean value correction factors for reducing the thermocouple temperatures to the optical scale are too large by five to seven degrees, depending, of course, upon the temperature, above 1400° C.

TABLE I.

Series.	Number of Energy Curves.	Mean Value, C.	Remarks.
1911			
I	26	14469	Unpainted Marquardt porcelain radiator.
1912			
I	7	14463	Unpainted radiator used in 1911. Apparatus newly adjusted to determine the reproducibility.
II	14	14432	Radiator painted with chromium oxide.
III	7	14466	New unpainted Marquardt radiator.
IV	9	14446	Radiator painted with a mixture of the oxides of cobalt and chromium.
V	14	14476	Radiator repainted; optical path readjusted.
VIII	8	14461	New thermocouples; other adjustments the same as for the preceding series.
IX	9	14436	New water-cooled shutter, rest of apparatus unmodified.

Mean value. $C = 14456 \pm 4$ (94 energy curves). $A = 2911 \pm 1$.

of a number of high-temperature, energy curves to the series of 1911, which required the optical scale corrections.

The series of 1911 is the most accurate as regards low humidity while the series of 1912 excel in having the most improved methods and the longest experience in the work. The results of the two years work are given in Table I. Each value of C is the mean of the number of isothermal energy curves (94 in all) given in column 2, computed by eq. (2) published in the previous communication to this journal. The mean value is

$$C = 14456 \pm 4,$$

$$A = 2911 \pm 1.$$

In the complete paper it will be shown that the values obtained by Lummer and Pringsheim, if computed by the present methods would give a mean value of $C = 14{,}465 \pm 40$, and that the values published by Paschen, if similarly corrected would give a mean value of $C = 14{,}458 \pm 25$.

The most recent communication from Warburg and his associates[1] gives a mean value of $C = 14{,}374 \pm 4$. The mean value of the four sets of investigations weighted as follows (L and $P = 1$, $P = 3$, $W = 10$ and $C = 10$) gives a mean value of

$$C = 14{,}420 \text{ mikron deg.}$$

$$A = 2{,}905 \text{ mikron deg.}$$

The mean value of the most recent work by Warburg and by the writer is $C = 14{,}415$.

The value $C = 14{,}420$ is a convenient one to use. The difference in the values obtained by Warburg and by the writer indicates an uncertainty of

[1] Warburg, Leithaüser, Hupka, Müller, Sitzber. Akad. Wiss. Berlin, I., p. 35, 1913.

five to six degrees at 1750° C. It is rather remarkable that the older data (which were reduced by the present system of computation long before the inconsistencies among the various sets of the present observations were eliminated) are in close agreement with the present results.

WASHINGTON, D. C.,
March 5, 1913.

THE ACTINODIELECTRIC EFFECT OF SULPHUR.[1]

BY CHESTER A. BUTMAN.

THE actinodielectric effect is effectively a change in conductivity due to the influence of light. This term was first used by Lenard and Saeland[2] to describe an effect discovered by them while experimenting on the phosphorescent alkaline earth sulphides. The effect was further examined in detail by Oeder, Ramsauer and Hausser[3] for phosphorescent Ca-S-Bi. These last investigators showed that not only did red light give the effect, but it was a maximium with yellow, a minimium with green, with a rapidly increasing value in the ultra-violet.

On account of the previously discovered similarity between the electrical properties of sulphur and of the phosphorescent alkaline earth *sulphides*[4] it occurred to the writer to see whether sulphur was also actinodielectric. Hence the effect was discovered and reported upon in a brief note.[5] The apparatus originally used has been redesigned, and during a brief vacation was set up in the new Sloane Laboratory at Yale University. The previous results obtained were confirmed, *i. e.*, the deflection of the electrometer was in the direction of the applied field, an effect was obtained with red light, and that the magnitude, the effect depended to some extent on the intensity of the light. The light in every case was passed through glass before it entered the chamber. In addition it was found that with *no field*, and a good *vacuum*, a photoelectric effect of the sulphur was obtained. Hence, the apparatus affords a convenient means of investigating the relation between the photoelectric and the actinodielectric effects. Further experiments are in progress at M. A. C., Amherst, Mass.

It has recently come to my attention that Bates[6] has independently discovered an effect in sulphur which is the same as the actinodielectric effect. Bates found that with a field on the guard ring of an electroscope with sulphur insulation, that the leaves charged up with a sign depending on the sign of the field when light was thrown on the sulphur.

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Lenard and Saeland, Ann. des Phys., 28, pp. 476–502, 1909.

[3] Oeder, Ramsauer and Hausser, Ann. der Phys., pp. 445–454, 1911.

[4] C. A. Butman, Amer. Jour. Sci., 34, p. 139, 1912.

[5] *Loc. cit.*

[6] Le Radium, 8, 312, 1911.

Another experiment of interest is that of F. Schmidt.[1] Schmidt has shown that the dielectric constant of sulphur and of phosphorescent Ca-S-Bi vary in the same direction, and approximately by the *same amount*, with a change in temperature from − 140° C. to + 80° C.

Note.—In view of the interesting discussion of the foregoing paper by Professor Ernest Merritt and Professor W. S. Franklin it may be well to call attention to a paper by G. Szivessy and K. Schäfer, entitled "Über die Erhöhung des elektrischen Leitvermögens bei flüssigen Dielektrika durch Bestrahlung mit ultraviolettem Lichte," Ann. der Phys., p. 511, 1911. These experimenters found that the electrical conductivity of paraffine oil was increased by projecting ultra-violet light upon it.

The experiments of Stark and Steubing[2] are important in this connection. They showed that all organic substances which became *fluorescent* by the absorption of light were *photoelectric*, while those which absorbed light and were not fluorescent gave no photoelectric effect. See also Nichols and Merritt, "Studies in Luminescence," p. 162, Carnegie Institution of Washington, 1912.

A New Vacuum Gage of Extreme Sensitiveness.[3]

By Irving Langmuir.

At very low pressures the viscosity of gases is one of their most marked characteristics. This property is made use of in the new gage.

The gage consists of a rotating disk above which is suspended, by a quartz fiber, another disk carrying a mirror. The viscosity of the gas causes it to be set in motion by the lower disk and this motion produces a torque on the upper disk which can be measured in the usual way by a beam of light reflected from the mirror.

The rotating disk is made of thin aluminium and is attached to a steel or tungsten shaft mounted on jewel bearings and carrying a magnetic needle. The suspended disk is of very thin mica. The lower disk can be rotated easily at a speed of 10,000 r.p.m. by means of a rotating magnetic field produced outside of the bulb containing the two disks. This field is most conveniently obtained by a Gramme ring supplied with current at six points from a commutating device run by a motor. In this way the speed of the motor determines absolutely the speed of the disk, since the two revolve in synchronism. The speed of the disk may thus be varied at will from a few revolutions per minute up to 10,000 or more.

The sensitiveness of the gage is extremely great. At 1,000 r.p.m., with a scale at about 60 cm. distance, we obtain about 400 mm. deflection for .001 mm.

[1] P. Lenard, Über Lichtsummen bei Phosphoren, p. 9; Sitz. d. Heidelberges Akad. d. Wiss. Math. nat. Kl., 1912.

[2] Phys. Zeit., 9, pp. 481, 661, 1908; also J. Stark, Prinzipien der Atomdynamik, II., p. 216, 1911.

[3] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

of air. We find the deflection exactly proportional to the pressure below about .01 mm., proportional to the speed of the revolving disk and practically independent of the distance between the two disks. For different gases at the same pressure the deflections are proportional to the square root of the molecular weight. All these facts are in accord with the kinetic theory. At 10,000 r.p.m., one mm. deflection corresponds to .00000025 mm. There should therefore be no difficulty in detecting pressures as low as 10^{-7} mm.